Electrostatic Fields

Electrical Engineering I · Module 92315 · Chapter 1

From Fundamentals to AI Apocalypse · personal study guide

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Made by Thanate Amornjiraporn (Focus)

Charges at rest create a field. The field pushes on other charges, stores energy, and changes when matter fills the space. This chapter follows that chain from a single point charge to the capacitor.

Start from memory

Before reading, answer these six prompts without notes. A wrong guess is useful: it tells you what to look for while you read.

1. What changes when you measure the field E instead of the force F?
2. Why can the fields of several charges simply be added?
3. What does Gauss's law count when you integrate over a closed surface?
4. Why is the inside of a metal body field-free in electrostatics?
5. At a boundary between two insulators with no surface charge, which stays continuous: the normal part of D or the normal part of E?
6. What is the difference between a capacitor's capacitance and its stored energy?

Answers: see "Start from memory" under Answers at the end.

How to use this guide

Every section ends with a short recall check. Answer it with the guide closed before you move on: retrieving an idea fixes it far better than rereading it.

1. **Day 0 (two 45-minute blocks):** do the six prompts above, then work through 1.1–1.14. After each section, close the page and answer its recall check. Mark every miss.
2. **Day 1:** rebuild the derivations in Derive it cold on blank paper, every step. Compare, then redo only the ones that broke.
3. **Day 3:** mixed practice 1–18 without notes. Mark misses, read the section named in each answer, take a break, then redo only the marked problems. Stop when a round adds no new marks.
4. **Day 6 or 7:** mixed practice 19–38, then answer the Explain it questions aloud in full sentences. Cover the chapter map and say what each idea rests on.
5. **About two weeks later and before the exam:** do the two exam-style problems against the clock, mixed with problems from later chapters. Tick a self-check line only when you have done it cold on a later day.

When stuck: after about ten minutes, reveal only the next step of the answer, not the whole working. Later, redo the full problem from a blank page. Looking at a solution and thinking "I could have done that" is not the same as doing it.

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How the chapter fits together

Every idea in the chapter rests on others. Choose one to see what it is built on and why; the three on the left are measured, everything else follows. To practise, cover the map and say what each idea rests on.

Measured · what the chapter starts from

- **Charge** §1.1

  two kinds, whole multiples of e, conserved

  Rests on: nothing; it is observed. Like charges repel, unlike attract, so there are two kinds. Rubbing moves charge; it never creates it.

- **Coulomb's law** §1.2

  $F \propto Q_1 Q_2 / r^2$, along the line between them

  Rests on: nothing; it is measured. Two features carry the rest of the chapter: the force is central and depends only on r, and the power is exactly 2.

- **Superposition** §1.4

  forces (and fields) add as vectors

  Rests on: nothing; it is measured. The force of several charges on a probe is the vector sum of the single forces, in a homogeneous medium.

Defined · quantities built on them

- **Field strength** §1.3

  $E = F/Q$, in V/m

  Rests on: Coulomb's law, Superposition. Well defined only because Coulomb's law makes F proportional to the probe charge, so F/Q does not depend on the probe. Superposition gives the field of any arrangement.

- **Potential** §1.8

  $\varphi_e(\mathrm{P}) = -\int E \cdot \mathrm{d}s$ from the reference

  Rests on: Field strength, Path independence. A number per point exists only if the work does not depend on the path, so this definition needs path independence first.

- **Voltage** §1.9

  $U_{12} = \varphi_1 - \varphi_2$

  Rests on: Potential. A difference of potentials: the reference point cancels, so a voltage does not depend on where zero is.

- **Flux density** §1.10

  $D = \varepsilon E$, in $\mathrm{C/m^2}$

  Rests on: Field strength. A second field quantity that counts the source charge directly and takes the effect of matter into ε.

- **Capacitance** §1.13

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$C = Q/U$, in F

Rests on: Voltage, Gauss's law. Useful only because Q and U are proportional for a fixed geometry, which follows from Gauss's law and the linearity of everything above.

Derived · results that follow

- **Path independence** §1.8

  $\oint E \cdot \mathrm{d}s = 0$

  Rests on: Coulomb's law, Superposition, Field strength. A central force that depends only on r does no net work around a closed loop. Superposition extends this to any arrangement of charges.

- **Gauss's law** §1.10

  $\oiint D \cdot \mathrm{d}A = Q_{\text{enclosed}}$

  Rests on: Coulomb's law, Superposition, Flux density. Needs the exact $1/r^2$: field times sphere area is then the same on every sphere. Superposition extends it to any charge distribution.

- **Interface conditions** §1.11

  normal D jumps by σ_A, tangential E continuous

  Rests on: Path independence, Gauss's law. The normal part comes from Gauss's law on a flat pillbox, the tangential part from the closed-loop integral on a thin rectangle.

- **Conductors** §1.12

  $E = 0$ inside, one potential, E normal to the surface

  Rests on: Charge, Field strength. Free charges move whenever $E \neq 0$. In electrostatics nothing moves, so the field inside must be zero.

- **Capacitor formulas** §1.13

  $\varepsilon A/d$, $4\pi\varepsilon_0 ab/(b-a)$, $2\pi\varepsilon/\ln(b/a)$ per length

  Rests on: Gauss's law, Voltage, Capacitance. The recipe: put ±Q on the electrodes, D from Gauss, E = D/ε, integrate to U, divide.

- **Stored energy** §1.14

  $W_e = \frac{1}{2} C\, U^2 = \frac{1}{2} Q\, U$

  Rests on: Voltage, Capacitance. Charging moves each dq through the voltage present, u = q/C; integrating gives $Q^2/(2C)$.

Which tool for which question

If the question asks for...	reach for...
the force on a given charge	$F = Q\,E$ (find E first)
the field of a few point charges	Coulomb field of each, then vector sum
the field of a sphere, long cylinder or plane	Gauss's law with a matching surface
work to move a charge, or a potential	$W_e = Q(\varphi_{\text{end}} - \varphi_{\text{start}})$, $\varphi = -\int E \cdot \mathrm{d}s$
the field on the far side of a boundary	normal D and tangential E conditions
charge per volt, or stored energy	$C = Q/U$, $W_e = \frac{1}{2} C\, U^2$

Made by Thanate Amornjinaporn (Taccus)

1.1 Electric charge

The observation. Rub two glass rods with wool and they repel; two plastic rods repel too; a glass rod and a plastic rod attract. Mechanics cannot explain this, so a new property is introduced: electric charge. Because there is both attraction and repulsion, there must be two kinds. By convention glass gets +, plastic gets −.

> **In plain words:** charge is a property of matter, like mass, except that it comes in two kinds, + and −. Like charges push each other apart and unlike charges pull together. Charge is never created or destroyed, only moved, and it always comes in whole multiples of the smallest amount, the charge e of one proton.

Unit and elementary charge

$$1\,\mathrm{C} = 1\,\mathrm{A\,s}$$
$$e \approx 1.602 \times 10^{-19}\,\mathrm{C}$$
$$\text{proton } +e,\ \text{electron } -e,\ \text{neutron } 0$$

Read it as: "one coulomb is one ampere flowing for one second", and "e is about 1.602 × 10⁻¹⁹ coulomb". Every charge is a whole multiple of e. A neutral atom has as many protons as electrons.

- **Charge is separated, not created.** Rubbing glass moves electrons onto the wool, leaving the glass with an electron deficit (+). Plastic takes electrons from the wool and ends up with an excess (−).
- **Conservation:** the total charge of a closed system is constant. Charges only appear or disappear in equal and opposite pairs.

Near miss: a rubbed glass rod seems to have gained charge, but none was created: electrons moved from the glass to the wool, leaving the rod positive and the wool equally negative. The total charge is still zero.

Recall check

1. Why does the rubbing experiment force us to introduce exactly two kinds of charge?
2. A body carries -4.8×10^{-19} C. What happened to it at the level of electrons?

Answers: see "1.1 Electric charge" under Answers at the end.

Made

1.2 Coulomb's law

For two point charges (bodies much smaller than their distance), the force grows with each charge and falls with the square of the distance. The same $1/r^2$ appears in Newton's gravitation; the $4\pi r^2$ in the denominator is the area of a sphere of radius r.

> **In plain words:** Coulomb's law says how hard two point charges push or pull: the force grows with each charge and falls with the square of the distance between them. Like charges repel and unlike charges attract, along the line joining them.

Force on charge 2 caused by charge 1

$$F_2 = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}\, e_{\mathrm{r}}$$

$$e_{\mathrm{r}} = \frac{r_{\mathrm{P2}} - r_{\mathrm{P1}}}{r}$$

Read it as: "the force on charge 2 is $Q_1 Q_2$ over $4\pi\,\epsilon_0\,r^2$, in the direction e_r from charge 1 to charge 2". $\varepsilon_0 = 8.854 \times 10^{-12}$ A s/(V m); $1/(4\pi\,\varepsilon_0) \approx 8.988 \times 10^9$ V m/(A s). e_{r} points from the charge that causes the force to the charge that feels it.



like charges repel

Like charges push each other apart: the force on each points away from the other.



unlike charges attract

Unlike charges pull together, with equal and opposite forces along the line joining them.

- **Signs are built in.** If $Q_1 Q_2 > 0$ the force points along e_{r} (repulsion); if < 0 it points against it (attraction). Put the signed charges in and the direction comes out right.
- **Action = reaction:** the force on charge 1 has the same size and the opposite direction.

Worked example

$Q_1 = +4$ µC sits at the origin, $Q_2 = -1$ µC at (0, 0.3 m). Find the force on Q_2.

1. e_{r} points from Q_1 to Q_2: $e_{\mathrm{r}} = +e_{\mathrm{y}}$, $r = 0.3$ m.
2. Size: $8.988 \times 10^9 \times (4 \times 10^{-6})(1 \times 10^{-6})/0.09 \approx 0.40$ N.
3. The product $Q_1 Q_2$ is negative, so $F_2 \approx -0.40$ N e_{y}: toward Q_1, an attraction, as opposite signs should give.

Trap: r is the distance between the charges, not a position coordinate. Point charges are an idealisation; Coulomb's law in this form needs bodies much smaller than their distance.

Near miss: doubling the distance between two charges looks as if it should halve the force, but the force falls to a quarter: it depends on $1/r^2$, not on $1/r$.

Recall check

1. Which way does e_r point in Coulomb's law?
2. Both charges are tripled and the distance is tripled. What happens to the force?

Answers: see "1.2 Coulomb's law" under Answers at the end.

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1.3 Electric field strength

Coulomb's law describes a pair of charges but says nothing about the space around them. The field idea fixes that: a charge changes the surrounding space, and that change is felt by any probe charge placed there. A field assigns a physical quantity to every point in space; temperature is a scalar field, gravity and E are vector fields.

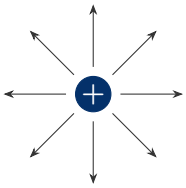
> **In plain words:** instead of asking how two charges act on each other, describe what one charge does to the space around it. The **field** E says, at every point, the force a charge of one coulomb would feel there; any charge Q placed at that point then feels the force $Q\,E$.

Field of one point charge Q at observation point P

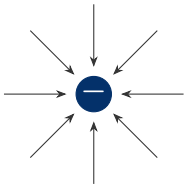
$$E(r_{\mathrm{P}}) = \frac{Q}{4\pi\,\varepsilon_0 r^2}\,e_{\mathrm{r}} = \frac{Q}{4\pi\,\varepsilon_0}\cdot\frac{r}{r^3}$$

$$r = r_{\mathrm{P}} - r_{\mathrm{Q}}$$

Read it as: "E at P is Q over $4\pi\,\epsilon_0\,r^2$ along e_r, pointing away from Q"; for a negative Q it points towards Q. Force on a probe: $F = Q_{\mathrm{probe}}\,E$. Unit of E: V/m (equivalent to N/C).



Positive charge: E points away from it in every direction and weakens as $1/r^2$.



Negative charge: E points towards it.

- E does not depend on the probe: a bigger probe charge feels a bigger force, but F/Q stays the same.
- The field exists in vacuum; no matter is needed.
- A positive probe is pushed along E, a negative probe against it.
- The vector r always runs from the source point Q to the observation point P.

Near miss: a bigger probe charge feels a bigger force, so the field might seem to depend on the probe, but $E = F/Q_{\mathrm{probe}}$ stays the same: double the probe and the force doubles with it.

Recall check

1. The probe charge is doubled. What happens to E and to F?
2. Write the field of a point charge in the r/r^3 form and say what r is.

Answers: see "1.3 Electric field strength" under Answers at the end.

Aparat Tiranae Amornjirapo

1.4 Superposition of fields

Experiments show that the forces of several charges on one probe add as vectors. In a homogeneous medium the total field is therefore the vector sum of the individual fields.

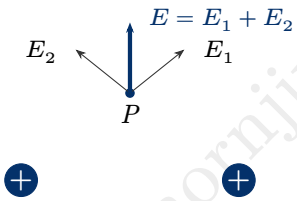
> **In plain words:** with several charges, each one makes its own field as if the others were not there, and the total field is their vector sum. Add the fields as arrows, component by component, not as plain numbers.

Field of *N* point charges

$$E(r_\mathrm{P}) = \frac{1}{4\pi\,\varepsilon_0}\sum_{\mathrm{i}=1}^{\mathrm{N}} Q_\mathrm{i}\,\frac{r_\mathrm{i}}{r_\mathrm{i}^3}$$

$$r_\mathrm{i} = r_\mathrm{P} - r_\mathrm{Qi}$$

Read it as: "the field at P is the sum over every charge of Q_i r_i over $4\pi\,\epsilon_0$ r_i^3": one term per charge.
The force on a further charge Q_4 at P is $Q_4\,E(r_\mathrm{P})$, where the sum leaves out Q_4 itself: a charge does not push on itself.



Superposition: at P the fields of the two charges add as arrows. Here their sideways parts cancel and the sum points straight up.

1. Sketch the charges and P with coordinates.
2. For each charge write $r_\mathrm{i} = r_\mathrm{P} - r_\mathrm{Qi}$ and its length.
3. Write each field with the signed charge.
4. Add x, y and z components separately; look for components that cancel by symmetry.

Worked example

Two charges of +2 nC sit at (0, +3 cm) and (0, −3 cm). Find E at P = (4 cm, 0).

1. $r_1 = (0.04, -0.03)$ m and $r_2 = (0.04, +0.03)$ m, both of length 0.05 m.
2. Each field has size $8.988 \times 10^9 \times 2 \times 10^{-9}/0.05^2 \approx 7.19 \times 10^3$ V/m along its own $r_\mathrm{i}/r_\mathrm{i}$.
3. The y-components (∓ 0.6 of each) cancel; the x-components (0.8 of each) add: $E \approx 2 \times 0.8 \times 7.19 \times 10^3 \approx 1.15 \times 10^4$ V/m along +x.

Trap: adding magnitudes. Two 7.19 kV/m fields gave 11.5 kV/m here, not 14.4 kV/m, because they point in different directions.

Thanue Amornjirapor

Near miss: $+Q$ and $-Q$ at equal distances on either side of a point P make fields of the same size there, yet they do not cancel: both point the same way, from the + charge towards the − charge, so at P they add.

Recall check

1. Under what condition does the notes' superposition formula hold?
2. Why does the field of the other charges, not the total field, act on a charge in the arrangement?

Answers: see "1.4 Superposition of fields" under Answers at the end.

1.5 Forces between charge distributions

The notes only pose two questions here, using two charged bodies V_1 and V_2. Both follow from what you already know:

In plain words: for charged bodies rather than points, cut each body into tiny pieces, treat every piece as a point charge, and add up the forces. The adding up becomes an integral over the volume; the hard part is knowing the field that the other body produces.

1. **Forces inside one body.** Every pair of charges inside the body pushes or pulls on each other with equal and opposite forces, so these internal forces add to zero for the body as a whole. They still matter: they decide where charge settles. In a conductor, for example, excess charge spreads out to the surface.
2. **Forces between two bodies.** Cut V_2 into small elements $\mathrm{d}Q = \varrho\,\mathrm{d}V$, find the field E_1 that V_1 alone produces at each element, and add up $\mathrm{d}F = E_1\,\mathrm{d}Q$. The force on V_1 is equal and opposite.

Force on body 2 from body 1

$$F_2 = \iiint_{\mathrm{V2}} \varrho_2\, E_1\, \mathrm{d}V$$

Read it as: "the force on body 2 is the integral over its volume of $\varrho_2\ E_1\ dV$": charge density times the field of body 1, added over every piece.
This applies $F = Q\,E$ element by element. The hard part is finding E_1, which is what the rest of the chapter is about.

Near miss: treating two charged metal spheres as point charges at their centres works while they are far apart, but close together their charges shift towards or away from each other on the metal, and the point-charge force is no longer accurate.

Recall check

1. How do you find the force that one charged body exerts on another?
2. When can two charged metal spheres not be treated as point charges at their centres?

Answers: see "1.5 Forces between charge distributions" under Answers at the end.

1.6 Charge densities

A point charge is an idealised charge squeezed into a single point, so its density would be infinite. Real charge is spread out, and you describe it with a density defined as a limit: take a small piece of line, area or volume around a point, divide its charge by its size, and let the piece shrink onto the point.

> **In plain words:** real charge is spread out, so you describe how much of it sits on each unit of length, area or volume: the **charge densities** λ, σ_A and ϱ_V. The total charge is the density added up over the line, surface or body.

Distribution	Density	Uniform case	Total charge	Unit
Line	$\lambda = \mathrm{d}Q/\mathrm{d}l$	$\lambda = Q/l$	$Q = \int_\mathrm{l} \lambda \, \mathrm{d}l$	C/m
Surface	$\sigma_\mathrm{A} = \mathrm{d}Q/\mathrm{d}A$	$\sigma_\mathrm{A} = Q/A$	$Q = \iint_\mathrm{A} \sigma_\mathrm{A} \, \mathrm{d}A$	C/m²
Volume	$\varrho_\mathrm{V} = \mathrm{d}Q/\mathrm{d}V$	$\varrho_\mathrm{V} = Q/V$	$Q = \iiint_\mathrm{V} \varrho_\mathrm{V} \, \mathrm{d}V$	C/m³

A surface charge is idealised as infinitely thin. Pick the volume element that matches the shape (appendix 7.3 of the notes): $\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ (Cartesian), $\rho\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z$ (cylinder), $r^2 \sin\vartheta\,\mathrm{d}r\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$ (sphere). If ϱ depends only on r, use thin shells: $\mathrm{d}V = 4\pi r^2\,\mathrm{d}r$.

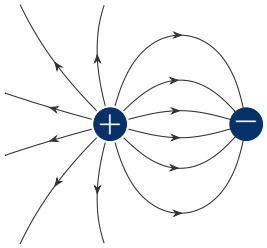
> **Worked example**
> A sphere of radius R has $\varrho(r) = \varrho_0(1 - r/R)$. Find its total charge.
>
> 1. The density depends only on r, so use shells: $Q = \int_0^\mathrm{R} \varrho_0(1 - r/R)\, 4\pi r^2 \,\mathrm{d}r$.
> 2. $= 4\pi\varrho_0 \,(R^3/3 - R^3/4) = \pi\varrho_0 R^3/3$.
> 3. Check: a uniform ϱ_0 would give $4\pi\varrho_0 R^3/3$. Our density falls to zero at the edge, so a quarter of that is plausible.

Near miss: stretching a uniformly charged rod to twice its length while keeping the same total charge halves its line density λ: a density is charge per length, not the total charge.

> **Recall check**
>
> 1. Give the three density symbols with their units.
> 2. When can you write $Q = \sigma_\mathrm{A} A$, and what must you do otherwise?
>
> *Answers: see "1.6 Charge densities" under Answers at the end.*

1.7 Field lines

A field line is a curve whose tangent at every point has the direction of E. Arrows on the line show the direction. In vector plots the arrow length shows the field strength; in line plots the density of lines does.



A dipole: field lines start on $+q$ and end on $-q$. They never cross, and they crowd together where the field is strong.

> **In plain words:** field lines are a picture of E: at every point the line runs in the direction of the field, starting on positive charges and ending on negative ones. Where the lines crowd together the field is strong; where they spread out it is weak.

1. Construction: find E at a starting point.
2. Step a short distance in that direction and find E again.
3. Repeat. The field line is the curve that is tangent to all these vectors.

- Lines start on positive charges and end on negative charges (or at infinity).
- Lines meet conductor surfaces at right angles (shown in 1.8 and 1.11).
- Lines never cross where $E \neq 0$: a point cannot have two field directions. They can only meet at field-free points.

Patterns to sketch from memory: a positive point charge (radial, outward), a negative one (radial, inward), the ideal plate capacitor (parallel lines from + to −), a + and − pair (lines bend from one charge into the other), and two equal + charges (lines push apart, with a field-free point exactly between them).

Uniqueness theorem: a field that satisfies Maxwell's equations and the boundary conditions is the only possible solution. Two arrangements with the same boundary conditions therefore have the same field. This is the deeper reason field lines cannot cross.

Near miss: field lines look like the paths charges would follow, but they are not: they show the direction of the force at each point, and a moving charge carried by its momentum can cut across them. Field lines also never cross, because E has one direction at each point.

> **Recall check**
>
> 1. Why is the midpoint between two equal positive charges field-free, and why can field lines meet only there?
> 2. State four rules for reading a field-line diagram.
>
> *Answers: see "1.7 Field lines" under Answers at the end.*

1.8 Electrostatic potential

Work. Moving a charge Q from P_0 to P_1 against the field costs energy; moving it with the field releases energy. The notes count the work supplied from outside, hence the minus sign.

> **In plain words:** moving a charge against the field takes work, like carrying a ball uphill. The **potential** φ_e is that work per coulomb, measured from a chosen reference point. Because the electrostatic field is conservative, the work depends only on where you start and finish, not on the path between.

Externally supplied work

$$W_\mathrm{e} = -\int_{\mathrm{P0}}^{\mathrm{P1}} F \cdot \mathrm{d}s = -Q \int_{\mathrm{P0}}^{\mathrm{P1}} E \cdot \mathrm{d}s$$

Read it as: "W_e is minus the integral of $F \cdot ds$ from P_0 to P_1": the work you supply against the field.
$W_\mathrm{e} > 0$: energy is supplied to the system. $W_\mathrm{e} < 0$: the field does the work and the system gives energy away.

Path independence. Carry the charge from P_0 along one path to P_1 and back along another. Afterwards the field is exactly as before, so the system's energy is unchanged and the total work is zero. Since the return path was arbitrary, the work from P_0 to P_1 cannot depend on the path.

The electrostatic field is conservative

$$\oint_\mathrm{C} E \cdot \mathrm{d}s = 0 \quad \text{and therefore} \quad \oint_\mathrm{C} F \cdot \mathrm{d}s = 0$$

Read it as: "the integral of $E \cdot ds$ round any closed path C is 0": going round and back costs no net energy.
The notes call it a source field, like gravity: the charges are its sources.

Potential. Because the work depends only on the end points, every point P can be given a number: the work per unit charge needed to bring a charge there from a reference point P_0, where $\varphi_\mathrm{e}(\mathrm{P}_0) = 0$.

Potential (unit V)

$$\varphi_\mathrm{e}(\mathrm{P}) = \frac{W_\mathrm{e}(\mathrm{P})}{Q} = -\int_{\mathrm{P0}}^{\mathrm{P}} E \cdot \mathrm{d}s, \qquad W_\mathrm{e} = Q(\varphi_\mathrm{e}(\mathrm{P}_\mathrm{end}) - \varphi_\mathrm{e}(\mathrm{P}_\mathrm{start}))$$

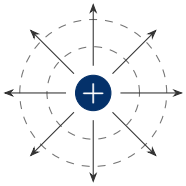
Usual references: earth, infinity, or circuit ground (the negative battery pole or a metal housing). Like height above sea level, only differences are physical.

Point charge Q_0, reference at infinity

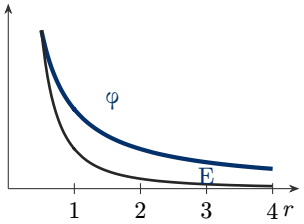
$$W_\mathrm{e} = \frac{Q\,Q_0}{4\pi\,\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad (\text{from } r_1 \text{ to } r_2)$$

$$\varphi_\mathrm{e}(r) = \frac{Q_0}{4\pi\,\varepsilon_0 r}$$

Read it as: "φ_e at distance r is Q_0 over $4\pi\,\epsilon_0\,r$": it falls as $1/r$, more slowly than the field.
Potential is a scalar: for several charges add the signed potentials, no components needed. $E \propto 1/r^2$, $D \propto 1/r^2$, $\varphi \propto 1/r$.



Equipotentials (dashed) of a point charge are spheres round it, and they cross every field line at a right angle.



Potential and field, scaled to meet at the smallest r: φ_e (blue) falls as $1/r$, E (black) as $1/r^2$, so the field dies away faster.

Move Q ...	same signs ($Q\ Q_0 > 0$)	opposite signs ($Q\ Q_0 < 0$)
toward Q_0 ($r_2 < r_1$)	$W_e > 0$: you push against repulsion	$W_e < 0$: attraction does the work
away from Q_0 ($r_2 > r_1$)	$W_e < 0$: repulsion does the work	$W_e > 0$: you pull against attraction
at constant distance	$W_e = 0$	$W_e = 0$ (a sphere around Q_0 is an equipotential)

Equipotential surfaces

An equipotential surface collects all points of equal potential; in a drawing its cuts appear as equipotential lines. Moving along one costs no work, so $E \cdot \mathrm{d}s = 0$ there: equipotentials cross field lines at right angles. Drawn at equal potential steps, closely spaced equipotentials mean a strong field.

Conductors. A tangential field at a metal surface would push the free charges along it, and they would keep moving until that component vanished. In the static state the whole conductor therefore sits at one potential, its surface is an equipotential, field lines meet it at right angles, and the interior is field-free.

Worked example

How much work is needed to bring +1 nC from infinity to 0.10 m from a fixed +2 nC?

1. φ at 0.10 m: $8.988 \times 10^9 \times 2 \times 10^{-9}/0.10 \approx 180$ V; $\varphi(\infty) = 0$.
2. $W_e = Q(\varphi_{end} - \varphi_{start}) = 10^{-9} \times 180 \approx 1.8 \times 10^{-7}$ J.
3. Positive, as the table predicts for pushing like charges together.

Near miss: the potential of a point charge falls as $1/r$ and its field as $1/r^2$, so at twice the distance the potential is half as large but the field only a quarter. Mixing up the two powers is the most common slip in this section.

Recall check

1. Explain in two sentences why $\oint E \cdot \mathrm{d}s = 0$ in electrostatics.
2. A negative charge is moved away from a positive charge. Is W_e positive or negative?
3. Why are equipotentials perpendicular to field lines?

Answers: see "1.8 Electrostatic potential" under Answers at the end.

1.9 Voltage

Write $\varphi_e(P_1) - \varphi_e(P_2)$ with the definition of potential. The two integrals from the reference point P_0 join into one integral from P_1 to P_2, and P_0 drops out: a potential difference does not depend on the reference point.

> **In plain words:** voltage is a difference of potentials: $U_{12} = \varphi_1 - \varphi_2$, the work per coulomb the field does carrying a charge from P_1 to P_2. The reference point drops out, so a voltage, unlike a potential, needs no agreed zero.

Voltage from P_1 to P_2 (unit V)

$$U_{12} = \varphi_e(P_1) - \varphi_e(P_2) = \int_{P1}^{P2} E \cdot \mathrm{d}s$$

Read it as: "U_{12} is φ_1 minus φ_2, the integral of $E \cdot ds$ from P_1 to P_2": first point minus second point.
Any path from P_1 to P_2 gives the same value. Uniform field along a path of length d: $U = E\,d$.

- $U_{12} > 0$ means P_1 is at the higher potential. E points from high to low potential.
- Link to work: moving Q from P_1 to P_2 needs $W_e = Q(\varphi_2 - \varphi_1) = -Q\,U_{12}$.

Sign check: U_{12} is "first point minus second point": $\varphi_1 - \varphi_2$. Read the subscripts in that order every time.

Near miss: a bird on a high-voltage line sits at a very high potential, yet it is safe, because the voltage between its two feet is almost zero. What matters is the difference, not the potential itself.

Recall check

1. Show in one line why the reference point P_0 drops out of U_{12}.
2. $\varphi_A = 5$ V and $\varphi_B = 8$ V. What is U_{AB}, and which way does E roughly point?

Answers: see "1.9 Voltage" under Answers at the end.

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1.10 Electric flux density and Gauss's law

Two field quantities. E describes the effect (the force on charges). The flux density D describes the cause and makes the influence of matter explicit. For the linear materials in this course they are proportional.

> **In plain words:** the flux density D describes the field from the side of its sources, the charges, while E describes its effect on charges. **Gauss's law** says that the total flux of D out of any closed surface equals the charge inside it, whatever shape the surface has.

Flux density (unit A s/m² = C/m²)

$$D = \varepsilon E = \varepsilon_0 \varepsilon_\mathrm{r} E$$

Read it as: "D is ϵ times E, that is ϵ_0 ϵ_r E".
$\varepsilon_\mathrm{r} = 1$ in vacuum and, to a good approximation, in air.

Electric flux through a surface (unit A s = C)

$$\Psi = \iint_\mathrm{A} D \cdot \mathrm{d}A, \qquad \mathrm{d}A = n\,\mathrm{d}A$$

n is the unit normal of the surface element. Only the part of D along n contributes.

Language trap: "flux" and "flux density" are historical names. Nothing flows in an electrostatic field; do not confuse flux with current, where charges really move.

Made by Thanate Amornsiripanitch

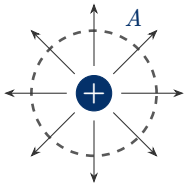
Gauss's law

Around a point charge in vacuum, $D = Q/(4\pi r^2)\ e_r$. On a concentric sphere D is parallel to $\mathrm{d}A$ and has constant size, so the flux is $D \times 4\pi r^2 = Q$, whatever the radius. The same holds for any closed surface and any position of the charge inside it, and by superposition for any charge distribution:

Gauss's law in integral form

$$\Psi = \oiint_{\mathrm{A}} D \cdot \mathrm{d}A = Q_{\mathrm{enclosed}} = \iiint_{\mathrm{V}} \varrho_{\mathrm{V}}\, \mathrm{d}V$$

Read it as: "the flux of D out of the closed surface A equals the charge enclosed".
Valid always, not only in electrostatics. In this course the material is described by ε, so Q means the charges you place (on electrodes or given as densities). The material's own polarisation charges are already included through ε.



A Gaussian surface A (dashed) round a charge: every field line leaving the charge crosses A once, so the flux through A equals the enclosed charge, whatever the shape of A.

Gauss's law always holds, but it only gives you the field when symmetry tells you the direction of D and that its size is constant on the surface. In practice that means three shapes: sphere (point charge, charged shell or ball), infinitely long cylinder (line charge, coax) and infinite plane (plate capacitor).

1. From the symmetry, state the direction of D and which coordinate it can depend on.
2. Choose a closed surface on which D is either perpendicular and constant, or parallel to the surface (zero flux).
3. Find the enclosed charge as a function of the surface size.
4. Set flux = enclosed charge, solve for D, then $E = D/\varepsilon$.

Worked example from the notes: long uniformly charged cylinder
Radius a, uniform ϱ_{V}. Use a coaxial cylinder of radius ρ and length l.

1. By symmetry $D = D_\rho(\rho)\ e_\rho$. On the end caps $e_\rho \cdot (\pm e_z) = 0$, so only the curved side counts: flux $= D_\rho\ 2\pi\rho l$.
2. Enclosed charge: $\varrho_{\mathrm{V}}\ \pi\ \rho^2\ l$ inside ($\rho < a$), $\varrho_{\mathrm{V}}\ \pi\ a^2\ l$ outside ($\rho \geq a$).
3. $D_\rho = \varrho_{\mathrm{V}}\ \rho/2$ inside and $\varrho_{\mathrm{V}}\ a^2/(2\rho)$ outside: it rises linearly to $\varrho_{\mathrm{V}} a/2$ at the surface, then falls as $1/\rho$. $E = D/\varepsilon$.

Worked example: the same charge in a dielectric
A 1 nC point charge sits in oil with $\varepsilon_r = 2.5$. At $r = 0.10$ m, $D = 10^{-9}/(4\pi \times 0.01) \approx 7.96 \times 10^{-9}$ C/m², exactly as in vacuum, because Gauss's law only sees the charge. But $E = D/(2.5\ \varepsilon_0) \approx 360$ V/m instead of 899 V/m.

Near miss: a closed surface with zero total flux might seem to contain no charge at all, but it can hold $+Q$ and $-Q$ together: Gauss's law gives only the net charge inside.

Recall check

1. State Gauss's law in words, including the direction in which the flux is counted.
2. Why do the end caps contribute nothing in the cylinder example?
3. A charge is moved from vacuum into a dielectric. Which of D and E changes?

Answers: see "1.10 Electric flux density and Gauss's law" under Answers at the end.

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1.11 Interface conditions

At a boundary, split E and D into a normal part (along the unit normal n) and a tangential part (along a unit tangent t). Each part is fixed by one law:

> **In plain words:** where two materials meet, the field changes, but not arbitrarily. Split E and D at the boundary into the part across it and the part along it: the part of E along the boundary is the same on both sides, and so is the part of D across it, as long as no free charge sits on the boundary.

1. **Normal part from Gauss's law.** Take a flat pillbox that straddles the boundary and shrink its height $h \to 0$. Only the two faces contribute: $(D_{n2} - D_{n1})A =$ enclosed charge $= \sigma_A A$.
2. **Tangential part from $\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = \mathbf{0}$.** Take a thin rectangular loop across the boundary and shrink its width $h \to 0$. Only the two long sides of length l contribute: $(E_{t2} - E_{t1})l = 0$.
3. Get the remaining parts from $D = \varepsilon E$ on each side.

Boundary	Normal part	Tangential part
Two dielectrics, no surface charge	$D_{n1} = D_{n2}$, so $E_{n1}/E_{n2} = \varepsilon_2/\varepsilon_1$	$E_{t1} = E_{t2}$, so $D_{t1}/D_{t2} = \varepsilon_1/\varepsilon_2$
Surface charge σ_A in vacuum	$D_{n2} - D_{n1} = \sigma_A$, with n pointing from side 1 into side 2	$E_{t1} = E_{t2}$ and $D_{t1} = D_{t2}$
Conductor (1) to vacuum (2)	Inside $D = E = 0$; outside $D = \sigma_A n$, $E = (\sigma_A/\varepsilon_0)n$, n pointing out of the metal	$E_t = D_t = 0$

Refraction of field lines. The two parts change differently, so a field line bends at the boundary, much like light. Measuring the angle α from the normal, $\tan\alpha = E_t/E_n$, and the two conditions give $\tan\alpha_1/\tan\alpha_2 = \varepsilon_1/\varepsilon_2$: in the medium with the larger ε the line leans further from the normal. (This rule follows directly from the table; the notes only mention the analogy with light.)

> **Worked example**
> A field of 100 V/m in vacuum hits a dielectric with $\varepsilon_r = 4$ at 45° to the normal. There is no surface charge.
>
> 1. Split: $E_{n1} = E_{t1} = 100 \cos 45° \approx 70.7$ V/m.
> 2. Tangential continuous: $E_{t2} = 70.7$ V/m. Normal D continuous: $E_{n2} = 70.7/4 \approx 17.7$ V/m.
> 3. $E_2 = \sqrt{70.7^2 + 17.7^2} \approx 72.9$ V/m at $\arctan(70.7/17.7) \approx 76°$ from the normal. Check: $\tan 45°/\tan 76° = 1/4 = \varepsilon_1/\varepsilon_2$.

Ma ... for Antonij Kapcar (Focus)

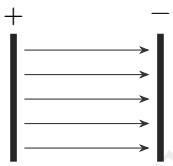
Ideal plate capacitor from two charged sheets

1. One infinite sheet with $+\sigma_A$: by symmetry D has the same size on both sides and points away from the sheet. The jump condition gives $D - (-D) = \sigma_A$, so $D = \sigma_A/2$ on each side.
2. The $-\sigma_A$ sheet gives $\sigma_A/2$ pointing toward it on both sides.
3. Superpose: outside the plates the two contributions cancel; between them they add to $D = \sigma_A$, directed from + to −.

Ideal plate capacitor (fringing neglected)

$$D = \sigma_A$$
$$E = \frac{\sigma_A}{\varepsilon}$$
$$U = E\,d$$

Read it as: "D equals the surface charge density σ_A; E is σ_A over ϵ; U is E times d".
Real plates have curved fringing fields at their edges. This course always neglects them.



uniform E between the plates

Plate capacitor: between the plates E is uniform and runs from + to −; the curved fringing field at the edges is neglected.

Near miss: at a boundary between air and glass, E might seem to keep its size across it, but only its part along the boundary does. Its part across the boundary drops by the factor ε_r of the glass, because it is the part of D across the boundary that stays equal.

Recall check

1. Which closed surface and which closed path do you use to derive the two interface conditions, and why must $h \to 0$?
2. Why is $E_t = 0$ just outside a conductor?
3. Field lines pass from a medium with $\varepsilon_r = 2$ into one with $\varepsilon_r = 8$. Do they bend toward or away from the normal?

Answers: see "1.11 Interface conditions" under Answers at the end.

1.12 Matter in an electrostatic field

The notes compare three versions of the same ideal plate capacitor. The plates carry a fixed charge $\pm Q$ (the capacitor is isolated), so $D = \sigma_A$ stays the same throughout. Only E, and therefore U, changes.

> **In plain words:** in an insulator the charges cannot flow away, but each atom or molecule stretches slightly in the field and becomes a tiny dipole. Their combined effect, the **polarisation** P, weakens the field inside the material by the factor ε_r; a metal goes further and cancels the field inside it completely.

Gap filled with	Field in the slab	Voltage
nothing (gap d)	—	$U = E\,d$, the largest
a dielectric slab (relative permittivity ε_r)	E/ε_r	in between
a metal slab of the same thickness	0	$E(d_1 + d_2)$, the smallest; only the air gaps count

Memory hook: empty gap > dielectric > metal. The notes name these voltages U_b and U_c differently on two pages (see Slips in the notes), so learn the order by material, not by letter.

Conductors: electrostatic induction

Put an uncharged metal body into a field. Its free electrons drift toward the positive plate, leaving a deficit on the far side. The body as a whole stays neutral, but its surfaces now carry opposite charges. If the body is made of two touching sheets that you pull apart while they are still in the field, each sheet keeps its charge. The charges stop moving once the whole body is one equipotential and the inside is field-free. Because these induced charges depend on the field they create, general field problems with conductors are hard to compute.

Hollow conductor. Put a charge Q at the centre of a metal shell with inner radius a and outer radius b. The inner surface collects $-Q$, the outer surface $+Q$. $D = Q/(4\pi r^2)$ in the cavity, it drops to zero inside the metal, and outside it looks like the field of the point charge again. At each surface the induced σ equals the D that arrives there, so D jumps by exactly σ, as the interface condition demands.

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Dielectrics: polarisation

An insulator has no free charge carriers, but its bound charges can shift or turn:

- **Displacement polarisation:** the field pulls an atom's electron cloud and nucleus slightly apart (electronic), or shifts ions in a crystal (ionic).
- **Orientation polarisation:** molecules with a built-in dipole, such as water, turn toward the field.

Dipole moment and polarisation

$$p = Q\,d \quad (d\,\text{from} - \text{to} +)$$

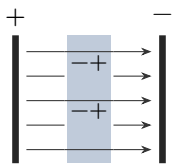
$$P = \frac{1}{V}\sum_{n=1}^{N} p_n$$

Read it as: "the dipole moment p is Q times d, with d pointing from $-$ to $+$"; "P is the total dipole moment per volume".
In a uniform field the two forces on a dipole are equal and opposite: no net force, but a torque that turns p toward E.

How polarisation enters D

$$D = \varepsilon_0 E + P$$
$$P = \varepsilon_0 \chi E$$
$$\chi = \varepsilon_r - 1$$
$$\text{so } D = \varepsilon_0 \varepsilon_r E$$

Read it as: "D is $\epsilon_0 E$ plus P", and with $P = \epsilon_0 \chi E$ that becomes $\epsilon_0 \epsilon_r E$.
χ is the electric susceptibility; vacuum and air: $\varepsilon_r = 1$, $\chi = 0$.



fewer E lines inside the slab

A dielectric slab in a charged capacitor: its molecules line up as small dipoles, and E inside is weaker by the factor ε_r, shown by fewer field lines. D is unchanged.

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The aligned dipoles leave a thin layer of bound charge on each face of the slab, opposite in sign to the neighbouring plate. These polarisation charges weaken the field inside. Read ε_r two ways: at the same D, E in the dielectric is ε_r times smaller; at the same E, D is ε_r times larger.

Worked example

An isolated plate capacitor has voltage $U_a = E\ d$ with an empty gap. A slab with $\varepsilon_r = 4$ now fills half the gap. Find the new voltage.

1. The charge is fixed, so D and the field E in the air stay the same; in the slab the field is $E/4$.
2. $U = E(d/2) + (E/4)(d/2) = (5/8)E\ d = 0.625\ U_a$.
3. A metal slab of the same thickness would give $0.5\ U_a$: lower still, as the hook predicts.

Near miss: sliding a dielectric slab into an isolated charged capacitor lowers its voltage, which can seem odd because no charge was removed: Q and D stay the same, but $E = D/\varepsilon$ falls, and with it $U = E\ d$.

Recall check

1. Order the three voltages (empty, dielectric, metal slab) for a capacitor with fixed charge, and justify the order in one sentence each.
2. What is the difference between induction and polarisation?
3. Which way does the dipole moment p point, and what does a uniform field do to a dipole?

Answers: see "1.12 Matter in an electrostatic field" under Answers at the end.

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1.13 Capacitors and capacitance

A capacitor is two conductors carrying equal and opposite charge, so that all the flux leaving the positive one ends on the negative one. Scale the charge by any factor and D, E and U scale by the same factor, so $Q \propto U$.

> **In plain words:** a capacitor stores charge on two conductors held apart. Put $+Q$ on one and $-Q$ on the other and a voltage U appears between them; the ratio $C = Q/U$ is the **capacitance**, the charge it holds per volt. It depends only on shape, size and material, not on how much charge is there.

Capacitance (unit A s/V = F)

$$C = \frac{Q}{U} = \frac{\oint\!\!\!\oint_{\mathrm{A}} D \cdot \mathrm{d}A}{\int_{\mathrm{s}} E \cdot \mathrm{d}s}$$

Read it as: "C is Q over U": the flux of D out of one electrode divided by the integral of E along a path to the other.
The surface encloses one electrode; the path runs from one electrode to the other. C depends only on geometry and material.

Recipe for any geometry: assume $\pm Q \to$ find D with Gauss $\to E = D/\varepsilon \to U = \int E \cdot \mathrm{d}s \to C = Q/U$. The Q must cancel; if it does not, there is a mistake.

Geometry	D between electrodes	Capacitance
Plates, area A, gap d	Q/A	$C = \varepsilon_0 \varepsilon_\mathrm{r} A/d$
Spherical, radii $a < b$, vacuum	$Q/(4\pi r^2)$	$C = 4\pi\ \varepsilon_0\ a\ b/(b-a)$
Coaxial, radii $a < b$, length l	$Q/(2\pi\rho l)$	$C/l = 2\pi\ \varepsilon/\ln(b/a)$

- **Spherical:** $U = Q/(4\pi\ \varepsilon_0)\ (1/a - 1/b) = Q(b-a)/(4\pi\ \varepsilon_0 a\ b)$. Limit check: for a thin gap $d = b - a \ll a$, $C \approx \varepsilon_0 4\pi a^2/d$, the plate formula with the sphere's area.
- **Coaxial:** $U = Q/(2\pi\ \varepsilon\ l) \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{d}\rho/\rho = Q\ \ln(b/a)/(2\pi\ \varepsilon\ l)$.
- **Large C in practice:** small gap (but the risk of breakdown grows), large area (metal foil on both sides of a thin film, rolled up: a wound capacitor), or high ε_r (electrolytic capacitor).

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Capacitor networks

Connection	Same for every capacitor	Total	Why
Parallel	voltage U	$C = \sum_{n=1}^{N} C_n$	charges add: $Q = U \sum_{n=1}^{N} C_n$
Series	charge Q	$1/C = \sum_{n=1}^{N} 1/C_n$	induction gives every plate pair $\pm Q$; voltages add

Quick checks: a parallel total is larger than the largest capacitor; a series total is smaller than the smallest. In series, the smallest capacitor takes the largest voltage. These are the reverse of the resistor rules.

> **Worked example**
> 6 µF and 3 µF in series across 12 V.
>
> 1. $1/C = 1/6 + 1/3 = 1/2$, so $C = 2$ µF; $Q = 2$ µF × 12 V = 24 µC on each.
> 2. $U_6 = 24/6 = 4$ V and $U_3 = 24/3 = 8$ V. They add to 12 V, and the smaller capacitor carries more voltage.

Built-up capacitors

- **Multilayer:** n interleaved plates per terminal form $2n - 1$ equal capacitors in parallel: $C = (2n - 1)\,\varepsilon A/d$.
- **Variable (rotary) capacitor:** the overlap of rotor and stator is a ring sector of angle α, area $\alpha(r_a^2 - r_i^2)/2$, so $C = (2n - 1)\,\varepsilon_0\alpha(r_a^2 - r_i^2)/(2d)$. Turning the rotor changes α and hence C.
- **Stray capacitance:** any two conductors form a capacitor. A three-phase overhead line has partial capacitances between every pair of wires and from each wire to earth.

Near miss: doubling the charge on a capacitor does not double its capacitance: U doubles too, so $C = Q/U$ stays the same.

> **Recall check**
>
> 1. Recite the five-step recipe for finding C.
> 2. Why do series capacitors carry the same charge?
> 3. What does the spherical formula reduce to when the gap is very thin?
>
> *Answers: see "1.13 Capacitors and capacitance" under Answers at the end.*

1.14 Energy stored in an electric field

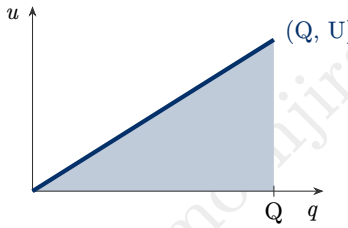
Charge a capacitor step by step. When it already holds $\pm q$, its voltage is $u = q/C$. Carrying a further $\mathrm{d}q$ from the negative to the positive plate, against the field, costs $\mathrm{d}W_\mathrm{e} = u\,\mathrm{d}q = (q/C)\,\mathrm{d}q$. (Lower-case letters mark quantities that change with time.)

> **In plain words:** charging a capacitor takes work, because each extra bit of charge is pushed against the voltage already there. That work is stored in the field between the plates and comes back out when the capacitor discharges.

Energy of a charged capacitor

$$W_\mathrm{e} = \frac{1}{C}\int_0^\mathrm{Q} q\,\mathrm{d}q = \frac{Q^2}{2C} = \tfrac{1}{2}\,C\,U^2 = \tfrac{1}{2}\,Q\,U$$

Read it as: "W_e is Q^2 over $2C$, which equals $\frac{1}{2}$ C U^2 and $\frac{1}{2}$ Q U".
Not Q U: the voltage grows from 0 to U during charging, so on average each bit of charge is lifted through only $U/2$.



Charging: the voltage $u = q/C$ grows from 0 to U as charge goes on, so the work is the triangle under the line: $\frac{1}{2}$ Q U.

Where is the energy? In the field. For a plate capacitor, $\frac{1}{2}C\,U^2 = \frac{1}{2}(\varepsilon A/d)(E\,d)^2 = \frac{1}{2}\varepsilon E^2 \times A\,d$, and $A\,d$ is the volume of the field region. Dividing by the volume gives an energy density that also holds for non-uniform fields:

Energy density (unit J/m³)

$$w_\mathrm{e} = \tfrac{1}{2}\,E\cdot D = \tfrac{1}{2}\,\varepsilon\,E^2$$
$$W_\mathrm{e} = \iiint_\mathrm{V} w_\mathrm{e}\,\mathrm{d}V$$

Read it as: "the energy per volume is $\frac{1}{2}$ $E\cdot D$, that is $\frac{1}{2}$ ϵ E^2".

Spring analogy: a spring with constant c_s stores $\frac{1}{2}c_\mathrm{s}X^2 = \frac{1}{2}{F_\mathrm{X}}^2/c_\mathrm{s}$. Matching it to $\frac{1}{2}Q^2/C$, capacitance plays the part of $1/c_\mathrm{s}$ (a soft spring) and voltage the part of force.

> **Worked example: two routes, one answer**
> Plates: $A = 0.01$ m², $d = 0.5$ mm, $\varepsilon_\mathrm{r} = 3$, $U = 50$ V.
>
> 1. Circuit route: $C = 3 \times 8.854 \times 10^{-12} \times 0.01/(5 \times 10^{-4}) \approx 531$ pF; $W_\mathrm{e} = \frac{1}{2} \times 531\ \mathrm{pF} \times 50^2 \approx 0.66$ µJ.
> 2. Field route: $E = U/d = 10^5$ V/m; $w_\mathrm{e} = \frac{1}{2} \times 3\varepsilon_0 \times 10^{10} \approx 0.133$ J/m³; volume 5×10^{-6} m³; $W_\mathrm{e} \approx 0.66$ µJ.

Near miss: doubling the voltage on a capacitor doubles its charge, but it quadruples the stored energy, because $W_\mathrm{e} = \frac{1}{2}\ C\ U^2$ grows with the square of U.

Recall check

1. Derive $W_\mathrm{e} = Q^2/(2C)$ in three lines.
2. Write the energy density two ways and give its unit.

Answers: see "1.14 Energy stored in an electric field" under Answers at the end.

Formula map and units

Close the page and rebuild this table from memory before checking it.

Quantity	Core relation	SI unit
Charge	$Q = N\ e$, $Q = \iiint \varrho_\mathrm{V}\ \mathrm{d}V$	C = A s
Force	$F = Q\ E$	N
Field strength	$E = Q\ r/(4\pi\ \varepsilon_0 r^3)$	V/m = N/C
Potential	$\varphi_\mathrm{e}(\mathrm{P}) = -\int_{\mathrm{P0}}^{\mathrm{P}} E \cdot \mathrm{d}s$	V
Voltage	$U_{12} = \varphi_1 - \varphi_2 = \int_1^2 E \cdot \mathrm{d}s$	V
Work (external)	$W_\mathrm{e} = Q(\varphi_\mathrm{end} - \varphi_\mathrm{start})$	J
Flux density	$D = \varepsilon_0\varepsilon_\mathrm{r}E = \varepsilon_0 E + P$	C/m²
Flux	$\Psi = \iint D \cdot \mathrm{d}A$; closed: $\Psi = Q_\mathrm{enc}$	C
Dipole moment, polarisation	$p = Q\ d$, $P = \varepsilon_0\chi E$, $\chi = \varepsilon_\mathrm{r} - 1$	C m, C/m²
Capacitance	$C = Q/U$	F = A s/V
Energy, energy density	$W_\mathrm{e} = \frac{1}{2}C\ U^2$, $w_\mathrm{e} = \frac{1}{2}E \cdot D$	J, J/m³

Check every answer: the units must match the quantity. Signs must agree with a sketch: attraction or repulsion, field away from + and toward −, work positive when you push against the field. Limits must make sense: fields fall off far away, piecewise formulas agree at the boundary, and a thin spherical gap behaves like plates.

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Derive it cold

Rebuild each derivation on blank paper, every step, without looking. The section number tells you where to check.

1. From Coulomb's law to the field of a point charge, and to the superposition sum. §1.3–1.4
2. Path independence from a round trip, and $\oint E \cdot \mathrm{d}s = 0$. §1.8
3. The potential of a point charge with the reference at infinity, including the sign discussion. §1.8
4. Why U_{12} does not depend on the reference point. §1.9
5. Gauss's law for a point charge on a sphere, and $D_\rho(\rho)$ for the charged cylinder. §1.10
6. Both interface conditions with a pillbox and a thin loop. §1.11
7. The ideal plate capacitor field from two charged sheets. §1.11
8. C of the plate, spherical and coaxial capacitor with the five-step recipe. §1.13
9. The series and parallel rules. §1.13
10. $W_\mathrm{e} = \frac{1}{2} C\, U^2$ by charging step by step, and $w_\mathrm{e} = \frac{1}{2} E\, D$ from the plate capacitor. §1.14

Explain it

Answer aloud in full sentences, as if teaching someone. If you stall, that is the gap to fix.

1. Why is the electrostatic field conservative, and what does that let you define?
2. Why must E be zero inside a conductor and perpendicular to its surface?
3. Why does a dielectric lower the voltage of an isolated charged capacitor, and why does a metal slab lower it even more?
4. When does Gauss's law give you the field, and when is it true but useless?
5. At an uncharged interface, why is the normal part of D continuous while the normal part of E jumps?
6. Why is a potential tied to a reference point, while a voltage is not?
7. Why do capacitors in series share the same charge?
8. Why is the energy of a capacitor $\frac{1}{2} Q\, U$ and not $Q\, U$?

Answers: see "Explain it" under Answers at the end.

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Mixed practice

Use $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m and $1/(4\pi\,\varepsilon_0) \approx 8.988 \times 10^{9}$ V m/(A s). The 38 problems are shuffled across sections, so first decide which idea a problem needs, then solve it. Each answer names the section to revisit if you miss it.

1. $Q_1 = +1$ µC sits at the origin and $Q_2 = +2$ µC at (3 cm, 4 cm). Find the force vector on Q_2.
2. A material has $\varepsilon_r = 4$ and $E = 250$ V/m. Find $|D|$.
3. A line density varies as $\lambda(x) = 2x$ µC/m from $x = 0$ to 3 m. Find the total charge.
4. $\varphi_A = 12$ V and $\varphi_B = -3$ V. Find U_{AB} and the work needed to move +2 µC from A to B.
5. A 10 µF capacitor is charged to 12 V. Find its stored energy.
6. At an uncharged interface, $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 4\varepsilon_0$, $E_{n1} = 100$ V/m and $E_{t1} = 80$ V/m. Find E_{n2} and E_{t2}.
7. Find the field magnitude 0.20 m from a −4 nC point charge and give its direction.
8. 2 µF and 8 µF are connected in series across 10 V. Find the charge and the voltage on each.
9. Find the potential 0.30 m from a +3 nC point charge, with zero potential at infinity.
10. An isolated plate capacitor has a 4 mm gap and 1000 V/m between the plates. Find the voltage (a) empty, (b) with a 1 mm uncharged metal slab inserted parallel to the plates, (c) with a 1 mm slab of $\varepsilon_r = 5$ instead.
11. An object carries $+4.806 \times 10^{-19}$ C. How many elementary charges is this, and what does the sign mean for its electrons?
12. Show that the two formulas for the charged cylinder give the same D at $\rho = a$. Why must they agree?
13. +3 nC sits at the origin and −3 nC at (6 cm, 0). Find E at P = (3 cm, 4 cm).
14. A vacuum region has a uniform field of 1.0×10^{5} V/m. Find the energy density.
15. A uniform 500 V/m field points from A to B, which are 0.08 m apart along the field. Find U_{AB}.
16. A spherical capacitor in vacuum has $a = 5$ cm and $b = 6$ cm. Find C and compare it with the thin-gap plate approximation.
17. A conducting surface in vacuum carries $\sigma_A = 4$ nC/m². Find the field just outside and inside the conductor.
18. A line carries a uniform 5 µC/m over 0.40 m. Find its charge.
19. −2 nC is moved from 0.20 m to 0.10 m from a fixed +5 nC. Find W_e and say who does the work.
20. Find the equivalent of 6 µF and 3 µF in parallel and in series, and say what is common to the elements in each case.
21. A uniform $D = 3$ µC/m² is normal to a 0.20 m² surface. Find the flux.
22. Find the force between +2 µC and −3 µC that are 0.20 m apart, and say whether it attracts or repels.
23. A 10 nF capacitor is charged to 100 V. A dielectric with $\varepsilon_r = 4$ then fills the gap. Find U, Q and W_e afterwards if (a) the source was disconnected first, (b) the source stays connected.
24. A long uniformly charged cylinder has $\varrho_V = 4$ µC/m³ and $a = 0.10$ m. Find D at $\rho = 0.05$ m.
25. A field of 200 V/m in a medium with $\varepsilon_r = 2$ meets a medium with $\varepsilon_r = 6$ at 30° to the normal. There is no surface charge. Find the size and angle of the field in medium 2.

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26. The distance between two point charges doubles. What happens to the force?

27. A multilayer capacitor has 5 plates per terminal, plate area 1 cm², layer thickness 10 μm and $\varepsilon_r = 1000$. Find C.

28. +2 μC moves 0.10 m along a uniform 200 V/m field. Find the externally supplied energy.

29. A ball of radius $R = 0.1$ m has $\varrho(r) = k\,r$ with $k = 10^{-6}$ C/m^4. Find its total charge and D at its surface.

30. +2 nC at $x = -0.10$ m and −2 nC at $x = +0.10$ m. Find E at the origin.

31. Plates with $\varepsilon_r = 2$, area 0.020 m², gap 1.0 mm. Find C.

32. A metal sphere of radius 0.10 m carries 2 nC. Find σ_A, the field just outside, and compare with the point-charge field at the same radius.

33. A closed surface encloses −8 nC. Find the outward flux of D.

34. −3 nC is brought from infinity to 0.20 m from a fixed +6 nC. Find W_e.

35. A uniform surface density of 8 nC/m² covers 0.25 m². Find the charge.

36. A coaxial cable has inner radius 1.0 mm, outer radius 4.0 mm and $\varepsilon_r = 2.5$. Find its capacitance per metre.

37. $+Q$ and $+2Q$ sit 30 cm apart. Where on the line between them is the field zero, and what does that point mean for the field-line picture?

38. A thin rod of length 0.5 m carries a uniform 2 nC/m and lies in a uniform external field of 4 kV/m. Find the force on the rod. How would you find it if the external field were not uniform?

Answers: see "Mixed practice" under Answers at the end.

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Exam-style problems

Allow about 20 minutes each, closed book. Each one chains several sections together.

A. Coaxial cable

Inner conductor radius 0.5 mm, outer conductor radius 2 mm, dielectric $\varepsilon_r = 2.2$, length 10 m, voltage 100 V.

1. Derive C/l with the five-step recipe, then find C.
2. Find the charge on the inner conductor.
3. Find E at the inner and the outer conductor. Where would the dielectric break down first?
4. Find the stored energy.

Answers: see "Exam-style problems" under Answers at the end.

B. Two-layer plate capacitor

Plate area 100 cm². Layer 1: 1 mm thick, $\varepsilon_r = 2$. Layer 2: 2 mm thick, $\varepsilon_r = 4$. The layers touch with no free charge between them, and 300 V is applied.

1. Which quantity is the same in both layers, and which interface condition says so?
2. Find D, E_1, E_2 and the voltage across each layer.
3. Find C in two ways: from Q/U, and as two capacitors in series.
4. Find the energy stored in each layer.

Answers: see "Exam-style problems" under Answers at the end.

Slips in the notes

We sent the professor a list of small slips in the WS 2026/27 notes, and the revised script (November 2026) fixes most of them. These three remain; page numbers are the ones printed on the pages.

- **p. 47:** not an error, but two notation switches to watch. Here Q_1 is the probe and Q the source (pp. 43–46 use Q and Q_0), and U stands for $\varphi(r_2) - \varphi(r_1)$, end minus start, the opposite order to $U_{12} = \varphi_1 - \varphi_2$ on p. 52. With that U, $W_e = Q_1 U$ is right; with the p. 52 convention the same work is $W_e = -Q\,U_{\text{start,end}}$.
- **p. 84 vs p. 90:** both figures now call the dielectric case (b) and the metal case (c), but the voltage names are swapped: pp. 84–87 call the metal-slab voltage U_b and the dielectric one U_c; p. 90 does the reverse. Physics: empty gap > dielectric > metal slab.
- **p. 92:** "ε_r describes the ratio χ" mixes two quantities. ε_r is the ratio of the field without and with dielectric at the same D; $\chi = \varepsilon_r - 1$ is the susceptibility. "Dieletric" is also misspelt. The professor agrees and plans to reword it.

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Self-check

Tick a skill only when you have done its problems without notes on a later day than you studied it. These are chapter learning goals, not an official exam specification.

☐ Explain charge sign, quantisation and conservation. (11)

☐ Use Coulomb's law as a vector with signed charges. (1, 22, 26)

☐ Tell field from force and find one from the other. (7)

☐ Add point-charge fields as vectors, using symmetry, and apply them to extended bodies. (13, 30, 38)

☐ Choose λ, σ or ϱ and integrate it, including non-uniform cases. (3, 18, 29, 35)

☐ Sketch and read field-line patterns and explain why lines cannot cross. (37)

☐ Derive the point-charge potential and get the sign of the work right. (9, 19, 28, 34)

☐ Use $U_{12} = \varphi_1 - \varphi_2$ and link it to work. (4, 15)

☐ Relate E, D, flux and Gauss's law with correct units. (2, 21, 33)

☐ Choose a Gaussian surface and solve for D. (12, 24, 29)

☐ Apply the interface conditions, including refraction and conductors. (6, 17, 25, 32)

☐ Explain induction and polarisation, and order the voltages for an empty, dielectric and metal-filled gap. (10, 23)

☐ Find C for plate, spherical, coaxial and multilayer capacitors, and combine them. (8, 16, 20, 27, 31, 36)

☐ Find capacitor energy and energy density by both routes. (5, 14, 23)

☐ Rebuild every derivation in Derive it cold on blank paper.

☐ Solve both exam-style problems in about 20 minutes each.

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Answers

Start from memory

1. E is a property of the space set up by the source charges; it does not depend on the probe. The force depends on the probe: $F = Q\,E$.
2. Experiments show that the forces from several charges on a probe add as vectors. In a homogeneous medium the fields therefore add as vectors too (superposition).
3. The total charge inside the surface: $\oint D \cdot \mathrm{d}A = Q_{\text{enclosed}}$.
4. Free charges move under any field inside the metal until their own field cancels it. In the static state nothing moves any more, so the interior field is zero.
5. The normal part of D. The normal part of E jumps in the ratio $\varepsilon_2/\varepsilon_1$.
6. Capacitance $C = Q/U$ depends only on geometry and material. Stored energy $\frac{1}{2}C\,U^2$ also depends on how far the capacitor is charged.

1.1 Electric charge

1. We see both attraction and repulsion; one kind of charge could only produce one of them.
2. $-4.8 \times 10^{-19}/1.602 \times 10^{-19} \approx -3$: it gained three electrons.

1.2 Coulomb's law

1. From the charge that produces the force toward the charge on which it acts.
2. Nothing: $3 \times 3/3^2 = 1$.

1.3 Electric field strength

1. E is unchanged; F doubles.
2. $E = Q\,r/(4\pi\,\varepsilon_0\,r^3)$, where $r = r_{\mathrm{P}} - r_{\mathrm{Q}}$ points from source to observation point.

1.4 Superposition of fields

1. In a homogeneous (linear) medium; that is where the fields of the individual charges simply add.
2. A charge exerts no net force on itself, so only the field of the other charges at its position counts.

1.5 Forces between charge distributions

1. Cut body 2 into small pieces, multiply each piece's charge $\varrho_2\,\mathrm{d}V$ by the field E_1 of body 1 at that piece, and add over body 2: $F_2 = \iiint \varrho_2\,E_1\,\mathrm{d}V$. §1.5
2. When they are close together: the charges on the metal shift towards or away from each other, so the charge is no longer spread as it would be on its own. §1.5

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1.6 Charge densities

1. λ in C/m, σ_A in C/m^2, ϱ_V in C/m^3.
2. Only for a uniform distribution. Otherwise keep the density inside the integral and integrate over the surface.

1.7 Field lines

1. The two fields there are equal in size and opposite in direction. Lines may only meet where $E = 0$, because anywhere else the direction is unique.
2. The tangent gives the direction of E; lines run from + to −; denser lines mean a stronger field; lines never cross where $E \neq 0$; lines meet conductors at right angles.

1.8 Electrostatic potential

1. After a round trip the field and the system's energy are exactly as before, so the net work is zero. Because this holds for any loop, the work between two points cannot depend on the path.
2. Positive: you pull against the attraction.
3. Moving along an equipotential costs no work, so $E \cdot \mathrm{d}s = 0$ for every step along it; E must be perpendicular.

1.9 Voltage

1. $\varphi_1 - \varphi_2 = -\int_{P0}^{P1} + \int_{P0}^{P2} = \int_{P1}^{P0} + \int_{P0}^{P2} = \int_{P1}^{P2} E \cdot \mathrm{d}s$.
2. $U_{AB} = -3$ V; E points from B (higher) toward A.

1.10 Electric flux density and Gauss's law

1. The outward flux of D through any closed surface equals the total charge enclosed.
2. D is radial and the cap normals point along $\pm z$, so $D \cdot \mathrm{d}A = 0$ there.
3. D stays the same (set by the charge); $E = D/(\varepsilon_0 \varepsilon_r)$ drops by a factor of ε_r.

1.11 Interface conditions

1. A thin pillbox (Gauss) for the normal part and a thin rectangle ($\oint E \cdot \mathrm{d}s = 0$) for the tangential part. With $h \to 0$ the side walls contribute nothing, leaving only the two faces or the two long sides.
2. A tangential field would move the free charges along the surface; in the static state they have already moved until it vanished.
3. Away from the normal: $\tan \alpha$ grows by the factor $8/2 = 4$.

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1.12 Matter in an electrostatic field

1. Empty is largest: the full gap has field E. Dielectric is in between: the field in the slab is reduced to E/ε_r. Metal is smallest: the field in the slab is zero.
2. Induction moves free charges across a conductor; polarisation only shifts or turns bound charges inside an insulator.
3. From the negative to the positive charge. A uniform field exerts no net force, only a torque that aligns p with E.

1.13 Capacitors and capacitance

1. Assume $\pm Q$; Gauss for D; $E = D/\varepsilon$; integrate E for U; $C = Q/U$.
2. Only the outer plates are charged from outside. Each inner pair of plates forms an isolated, neutral conductor, which induction splits into $-Q$ and $+Q$.
3. The plate capacitor with area $4\pi a^2$: $C \approx \varepsilon_0 4\pi a^2/d$.

1.14 Energy stored in an electric field

1. $\mathrm{d}W = u\,\mathrm{d}q$; $u = q/C$; $\int_0^{\mathrm{Q}} (q/C)\,\mathrm{d}q = Q^2/(2C)$.
2. $w_\mathrm{e} = \frac{1}{2} E \cdot D = \frac{1}{2}\varepsilon E^2$, in $\mathrm{J/m^3}$.

Explain it

1. After any round trip the field, and so the system's energy, is unchanged, so the net work is zero. The work between two points is therefore path-independent, which lets you assign each point a potential.
2. Free charges move under any field. In the static state they have stopped, so there is no field inside and no tangential component at the surface; only a normal component remains outside.
3. With the charge fixed, D is fixed. A dielectric reduces E in the slab to E/ε_r through polarisation charges; a metal reduces it to zero through induced charges. Less field over the same distance means less voltage.
4. When symmetry fixes the direction of D and makes its size constant on a closed surface (sphere, long cylinder, plane). Otherwise it still gives the total flux, but not the field at a point.
5. A flat pillbox with no enclosed charge has zero total flux, so the normal D going in equals the normal D coming out. Since $E = D/\varepsilon$ and ε differs on the two sides, the normal E must differ.
6. A potential is work per charge measured from a chosen zero point. In a difference of two potentials, the path from the reference cancels out.
7. Only the outer plates are connected to the source. Each inner pair of plates is an isolated, neutral conductor, so induction can only split it into $-Q$ and $+Q$.
8. While charging, the voltage rises from 0 to U. Each bit of charge is moved through the voltage present at that moment, which averages $U/2$.

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Mixed practice

1. $r = 0.05$ m, $|F| = 8.988 \times 10^9 \times 2 \times 10^{-12}/0.0025 \approx 7.19$ N along $(0.6, 0.8)$: $F \approx (4.31, 5.75)$ N, repulsion. §1.2
2. $D = \varepsilon_0 \varepsilon_r E \approx 8.85 \times 10^{-9}$ C/m². §1.10
3. $Q = \int_0^3 2x \, \mathrm{d}x$ μC $= 9$ μC. §1.6
4. $U_{AB} = 15$ V; $W_e = Q(\varphi_B - \varphi_A) = 2$ μC $\times (-15$ V$) = -30$ μJ: the field does the work. §1.9
5. $\frac{1}{2} \times 10$ μF $\times 12^2 = 720$ μJ. §1.14
6. $E_{n2} = (\varepsilon_1/\varepsilon_2) E_{n1} = 50$ V/m; $E_{t2} = 80$ V/m. §1.11
7. $8.988 \times 10^9 \times 4 \times 10^{-9}/0.04 \approx 899$ V/m, pointing toward the charge. §1.3
8. $C = 1.6$ μF, $Q = 16$ μC; $U_{2\mu F} = 8$ V, $U_{8\mu F} = 2$ V. §1.13
9. $8.988 \times 10^9 \times 3 \times 10^{-9}/0.30 \approx 89.9$ V. §1.8
10. (a) 4.0 V; (b) the field is zero in the metal: $1000 \times 0.003 = 3.0$ V; (c) $1000 \times 0.003 + 200 \times 0.001 = 3.2$ V. Empty > dielectric > metal. §1.12
11. $4.806 \times 10^{-19}/1.602 \times 10^{-19} = 3$: a deficit of three electrons. §1.1
12. Inside: $\varrho_V a/2$; outside: $\varrho_V a^2/(2a) = \varrho_V a/2$. There is no surface charge at $\rho = a$, so normal D must be continuous; a mismatch would reveal an algebra slip. §1.10
13. Both distances are 0.05 m, each field $\approx 1.08 \times 10^4$ V/m. E_1 points along $(0.6, 0.8)$; E_2 points toward the −3 nC, along $(0.6, -0.8)$. The y-parts cancel: $E \approx 1.29 \times 10^4$ V/m along +x. §1.4
14. $w_e = \frac{1}{2} \varepsilon_0 E^2 \approx 44$ mJ/m³. §1.14
15. $U_{AB} = \int_A^B E \cdot \mathrm{d}s = 500 \times 0.08 = +40$ V. §1.9
16. $C = 4\pi\ \varepsilon_0 \times 0.05 \times 0.06/0.01 \approx 33.4$ pF. The plate approximation $\varepsilon_0 4\pi a^2/d \approx 27.8$ pF is about 17 % low, because the gap (1 cm) is not small compared with a. §1.13
17. Outside $\sigma_A/\varepsilon_0 \approx 452$ V/m, normal to the surface; inside 0. §1.11
18. $Q = \lambda l = 2.0$ μC. §1.6
19. $W_e = Q\ Q_0/(4\pi\ \varepsilon_0)\ (1/0.10 - 1/0.20) = 8.988 \times 10^9 \times (-10^{-17}) \times 5 \approx -0.45$ μJ: attraction does the work. §1.8
20. Parallel 9 μF (same voltage); series 2 μF (same charge). §1.13
21. $\Psi = D\ A = 0.60$ μC. §1.10
22. $8.988 \times 10^9 \times 6 \times 10^{-12}/0.04 \approx 1.35$ N, attraction. §1.2
23. Before: $Q = 1$ μC, $W_e = 50$ μJ. Afterwards $C = 40$ nF. (a) Q stays 1 μC, $U = 25$ V, $W_e = 12.5$ μJ. (b) U stays 100 V, $Q = 4$ μC, $W_e = 200$ μJ. §1.12, 1.14
24. $D_\rho = \varrho_V \rho/2 = 4 \times 10^{-6} \times 0.05/2 = 1.0 \times 10^{-7}$ C/m². §1.10
25. $E_{n1} \approx 173$ V/m, $E_t = 100$ V/m; $E_{n2} = 173 \times 2/6 \approx 57.7$ V/m. $|E_2| \approx 115$ V/m at 60° from the normal ($\tan 30°/\tan 60° = 1/3 = \varepsilon_1/\varepsilon_2$). §1.11
26. A quarter, since $F \propto 1/r^2$. §1.2
27. $9 \times 8.854 \times 10^{-12} \times 1000 \times 10^{-4}/10^{-5} \approx 0.80$ μF. §1.13
28. $\Delta\varphi = -E\ d = -20$ V, so $W_e = Q\Delta\varphi = -40$ μJ: the field supplies the energy. §1.8

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29. $Q = \int_0^R k\, r\, 4\pi r^2\, \mathrm{d}r = \pi k\, R^4 \approx 3.14 \times 10^{-10}$ C; $D(R) = Q/(4\pi R^2) = k\, R^2/4 = 2.5 \times 10^{-9}$ C/m². §1.6, 1.10

30. Each field is $\approx 1.80 \times 10^3$ V/m toward +x (away from +, toward −): $E \approx 3.60 \times 10^3$ V/m along +x. §1.4

31. $C = \varepsilon_0 \varepsilon_r A/d \approx 354$ pF. §1.13

32. $\sigma_A = Q/(4\pi a^2) \approx 1.59 \times 10^{-8}$ C/m²; $E = \sigma_A/\varepsilon_0 \approx 1.80$ kV/m, the same as the point-charge field $8.988 \times 10^9 \times 2 \times 10^{-9}/0.01$. §1.11

33. $\Psi = Q_{\text{enclosed}} = -8$ nC (net flux points inward). §1.10

34. $8.988 \times 10^9 \times (-3 \times 10^{-9})(6 \times 10^{-9})/0.20 \approx -0.81$ µJ. §1.8

35. $Q = \sigma A = 2.0$ nC. §1.6

36. $2\pi \times 8.854 \times 10^{-12} \times 2.5/\ln 4 \approx 100$ pF/m. §1.13

37. At distance x from $+Q$: $Q/x^2 = 2Q/(0.30 - x)^2$, so $0.30 - x = \sqrt{2}\, x$ and $x \approx 12.4$ cm, closer to the smaller charge. It is the only point where field lines can meet; lines from both charges push apart around it. §1.7

38. Charge $Q = \lambda l = 1$ nC, so $F = Q\, E = 4$ µN along the field. In a non-uniform field, add up $\mathrm{d}F = \lambda\, E(x)\, \mathrm{d}x$ along the rod. §1.5

Exam-style problems

1. $D = Q/(2\pi\rho l)$, $U = Q \ln(b/a)/(2\pi\, \varepsilon\, l)$, $C/l = 2\pi\, \varepsilon_0 \varepsilon_r/\ln 4 \approx 88.3$ pF/m, so $C \approx 883$ pF.
2. $Q = C\, U \approx 88$ nC.
3. $E(\rho) = U/(\rho \ln(b/a))$: about 144 kV/m at $\rho = 0.5$ mm and 36 kV/m at $\rho = 2$ mm. The field is strongest at the inner conductor, so breakdown starts there.
4. $W_e = \frac{1}{2} C\, U^2 \approx 4.4$ µJ.

Exam-style problems

1. D: the field is normal to the interface and there is no surface charge, so $D_{n1} = D_{n2}$.
2. $U = D(d_1/\varepsilon_1 + d_2/\varepsilon_2)$, so $D = 300\ \varepsilon_0/(0.5\text{ mm} + 0.5\text{ mm}) \approx 2.66$ µC/m². $E_1 = 150$ kV/m and $E_2 = 75$ kV/m; 150 V across each layer.
3. $Q = D\, A \approx 26.6$ nC, $C = Q/U \approx 88.5$ pF. Series: $C_1 = C_2 \approx 177$ pF, so $C \approx 88.5$ pF.
4. $\frac{1}{2} \times 177$ pF $\times\, 150^2 \approx 2.0$ µJ per layer, 4.0 µJ in total, which matches $\frac{1}{2} C\, U^2$.

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