Exercise 2: Worked solutions

Electrical Engineering I · Module 92315 · Exercise 2

From Fundamentals to AI Apocalypse · personal study guide

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Try each part before opening the answer. The steps build fields and forces from direction and distance, then check against symmetry, units, or a limiting case.

Task 4: Electric force compared with gravity

Protons at A carry $Q_{\mathrm{A}} = +1$ C and electrons at B carry $Q_{\mathrm{B}} = -1$ C; they are 1 km apart. Find $|F_{\mathrm{E}}|/|F_{\mathrm{G}}|$. Use $\gamma = 6.667 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $e = 1.602 \times 10^{-19}$ C, $m_{\mathrm{p}} = 1.672 \times 10^{-27}$ kg and $m_{\mathrm{e}} = 9.1094 \times 10^{-31}$ kg.

Connected topic: Coulomb's law (§1.2) gives the electric force; compare it with the supplied gravitational law.

4: Magnitude ratio

Try first: convert each 1 C charge quantity into a number of elementary charges, then form both force magnitudes before dividing.

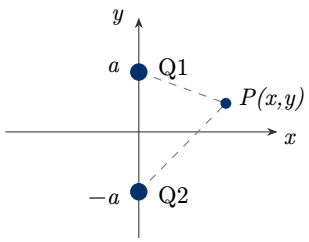
Answers: see "Task 4: Electric force compared with gravity" under Answers at the end.

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Task 5: Field from two charges on the y-axis

Place Q_1 at $(0,\ a)$ and Q_2 at $(0,\ -a)$. Find the field throughout the xy-plane and simplify it on the x-axis for the stated charge choices. With charges in coulombs and lengths in metres, field values are in N/C.

Connected topics: Electric field strength (§1.3) defines one source field; superposition (§1.4) adds them.



Resolve each contribution from its own source. P is arbitrary; its displacement from each charge sets direction and distance.

5a: Field anywhere in the plane

Try first: write the displacement from each source to P $= (x,\ y)$, then add both vectors.

Answers: see "Task 5: Field from two charges on the y-axis" under Answers at the end.

5b: Equal charges, $Q_1 = Q_2 = Q$

Try first: set $y = 0$ and identify which components cancel by symmetry.

Answers: see "Task 5: Field from two charges on the y-axis" under Answers at the end.

5c: Opposite charges, $Q_1 = Q$ and $Q_2 = -Q$

Try first: again set $y = 0$. Opposite signs change which components reinforce.

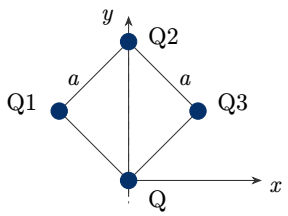
Answers: see "Task 5: Field from two charges on the y-axis" under Answers at the end.

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Task 6: Charges at the corners of a square

Four nonzero point charges occupy corners of a square of side a. Q is at the origin, Q_2 lies on the positive y-axis, and the surrounding permittivity is ϵ_0. With SI values, force is measured in newtons and work in joules.

Connected topics: Coulomb's law (§1.2) gives each force; superposition (§1.4) combines them. Work connects to electrostatic potential (§1.8).



The square is rotated in the coordinate plane. Sloping sides are a; the vertical diagonal Q to Q_2 is $\sqrt{2}a$.

6a: Force on Q at the origin

Try first: find the source coordinates from the diagram, then resolve each force into x and y components.

Answers: see "Task 6: Charges at the corners of a square" under Answers at the end.

6b: Condition for zero force

Try first: set both components from 6a to zero. The central charge Q is nonzero.

Answers: see "Task 6: Charges at the corners of a square" under Answers at the end.

6c: Work to move Q_2 to (0, a)

Try first: with $Q_1 = Q_3 = 0$, compare its initial and final distance from Q. Take work as external work for a quasistatic move.

Answers: see "Task 6: Charges at the corners of a square" under Answers at the end.

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Answers

Task 4: Electric force compared with gravity

4 · Coulomb force versus gravity

1. Gravity needs group masses, so first convert total charge to particle count. Each particle has charge magnitude e:

$$N = \frac{|Q|}{e}; \quad m_{\mathrm{A}} = \frac{|Q_{\mathrm{A}}|m_{\mathrm{p}}}{e}; \quad m_{\mathrm{B}} = \frac{|Q_{\mathrm{B}}|m_{\mathrm{e}}}{e}$$

2. Coulomb's law gives electric magnitude; opposite signs make the force attractive:

$$|F_{\mathrm{E}}| = \frac{k|Q_{\mathrm{A}}Q_{\mathrm{B}}|}{r_{\mathrm{AB}}^2}, \quad k = \frac{1}{4\pi\varepsilon_0}$$

3. Insert the group masses into the given gravity relation:

$$|F_{\mathrm{G}}| = \frac{\gamma m_{\mathrm{A}} m_{\mathrm{B}}}{r_{\mathrm{AB}}^2} = \frac{\gamma|Q_{\mathrm{A}}Q_{\mathrm{B}}|m_{\mathrm{p}}m_{\mathrm{e}}}{e^2 r_{\mathrm{AB}}^2}$$

4. Divide the magnitudes; the charge quantities and separation cancel:

$$\frac{|F_{\mathrm{E}}|}{|F_{\mathrm{G}}|} = \frac{k\,e^2}{\gamma m_{\mathrm{p}} m_{\mathrm{e}}}$$

5. Substitute $k = 8.9877 \times 10^9$ N m^2/C^2 and the given constants:

$$\frac{|F_{\mathrm{E}}|}{|F_{\mathrm{G}}|} \approx 2.27 \times 10^{39}$$

Result: electric force is about 2.27×10^{39} times stronger in magnitude.

For a feel of the sizes: $|F_{\mathrm{E}}| = k(1\ \mathrm{C})^2/(1000\ \mathrm{m})^2 \approx 8.99 \times 10^3$ N, roughly the weight of a 900 kg car, while the two particle groups weigh only $m_{\mathrm{A}} \approx 1.04 \times 10^{-8}$ kg and $m_{\mathrm{B}} \approx 5.69 \times 10^{-12}$ kg, so $|F_{\mathrm{G}}| \approx 3.96 \times 10^{-36}$ N.

Check: the ratio is dimensionless and independent of distance since both laws vary as $1/r^2$.

Task 5: Field from two charges on the y-axis

5a · General field by superposition

1. Point each displacement from source to P by subtracting source coordinates from point coordinates:

$$r_1 = x\,e_x + (y - a)e_y; \quad r_2 = x\,e_x + (y + a)e_y$$

2. A point-charge vector field is kQr/r^3; signed Q handles direction. Superpose the two source fields:

$$E(\mathrm{P}) = \frac{kQ_1 r_1}{|r_1|^3} + \frac{kQ_2 r_2}{|r_2|^3}$$

3. Take the lengths from the displacement components and substitute:

$$E(x, y) = \frac{kQ_1[x\,e_x + (y - a)e_y]}{[x^2 + (y - a)^2]^{3/2}} + \frac{kQ_2[x\,e_x + (y + a)e_y]}{[x^2 + (y + a)^2]^{3/2}}$$

4. **Result:** valid except at $(0, \pm a)$, where an ideal point-charge field is singular.

   Check: a positive source points along its source-to-P displacement; a negative source reverses it.

Task 5: Field from two charges on the y-axis

5b · Equal charges on the x-axis

1. At P = $(x, 0)$, the displacements have equal lengths and opposite vertical components:

$$r_1 = x\,e_x - a\,e_y; \quad r_2 = x\,e_x + a\,e_y; \quad |r_1| = |r_2| = \sqrt{x^2 + a^2}$$

2. With equal Q, vertical components cancel and horizontal components add:

$$E(x, 0) = \frac{kQ(r_1 + r_2)}{(x^2 + a^2)^{3/2}} = \frac{2kQxe_x}{(x^2 + a^2)^{3/2}}$$

3. **Result:** for positive Q it points +x when x > 0 and reverses for x < 0; negative Q reverses these directions.

   Check: at x = 0 symmetry requires zero, which the expression gives.

Task 5: Field from two charges on the y-axis

5c · Opposite charges on the x-axis

1. The distances still match, but signed contributions use the difference of the displacement vectors:

$$r_1 - r_2 = -2a\, e_y$$

2. Horizontal parts cancel; vertical contributions add downward:

$$E(x,0) = \frac{kQ(r_1 - r_2)}{(x^2 + a^2)^{3/2}} = -\frac{2kQae_y}{(x^2 + a^2)^{3/2}}$$

3. **Result:** for positive Q it points from upper positive to lower negative: −y. Negative Q reverses it.

   Check: at the midpoint both fields point down. Far along the axis, magnitude falls as $1/|x|^3$, as for a dipole.

Task 6: Charges at the corners of a square

6a · Vector sum at the origin

1. The square's 45° sides project equally on both axes. The other corners and their distances from Q are:

$$Q_1 : \left(-\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right); \quad Q_2 : (0, \sqrt{2}a); \quad Q_3 : \left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$$

   Side charges are distance a; Q_2 is distance $\sqrt{2}$a.

2. The top charge contributes along −y for positive QQ_2; its distance is $\sqrt{2}$a:

$$F_2 = -\frac{kQQ_2}{2a^2} e_y$$

3. The side-charge unit directions from each source toward the origin give:

$$F_1 = \frac{kQQ_1}{\sqrt{2}a^2}(e_x - e_y); \quad F_3 = -\frac{kQQ_3}{\sqrt{2}a^2}(e_x + e_y)$$

4. Add components and factor the scale, where $k = 1/(4\pi\epsilon_0)$:

$$F = \frac{kQ}{a^2}\left[\frac{Q_1 - Q_3}{\sqrt{2}}\, e_x - \left(\frac{Q_1 + Q_3}{\sqrt{2}} + \frac{Q_2}{2}\right) e_y\right]$$

5. **Result:** this vector is the net force on Q; charge signs select attraction or repulsion.

   Check: each term has charge product over distance squared. If $Q_1 = Q_3$, symmetry removes the x-component.

Task 6: Charges at the corners of a square

6b · Cancel both components

1. Since $Q \neq 0$, the shared factor kQ/a^2 cannot make the force zero. The x-component requires:

$$\frac{Q_1 - Q_3}{\sqrt{2}} = 0 \quad \Longrightarrow \quad Q_1 = Q_3$$

2. The y-component must also vanish:

$$\frac{Q_1 + Q_3}{\sqrt{2}} + \frac{Q_2}{2} = 0$$

3. Use $Q_3 = Q_1$ and solve:

$$Q_2 = -\sqrt{2}(Q_1 + Q_3) = -2\sqrt{2}Q_1$$

4. **Result:** $Q_1 = Q_3$ and $Q_2 = -2\sqrt{2}Q_1$.

   Check: equal side charges cancel horizontally; the top charge must have the opposite sign so its vertical force balances theirs.

Task 6: Charges at the corners of a square

6c · Change in electrostatic potential energy

1. The diagram places Q_2 at $(0, \sqrt{2}a)$; the requested endpoint is $(0, a)$. Only the interaction with Q remains. This starting position is read from the diagram.

2. For two point charges, $U = kQQ_2/r$. Quasistatic external work equals the potential-energy change:

$$W_{ext} = \Delta U = kQQ_2\left(\frac{1}{a} - \frac{1}{\sqrt{2}a}\right)$$

3. Use $k = 1/(4\pi\epsilon_0)$:

$$W_{ext} = \frac{QQ_2}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right)$$

4. **Result:** the signed external work is positive for $QQ_2 > 0$ and negative for $QQ_2 < 0$.

   If the task is read as bringing Q_2 in from far away ($r \to \infty$) instead of from its corner, the $1/(\sqrt{2}a)$ term becomes 0 and $W_{ext} = QQ_2/(4\pi\varepsilon_0 a)$. Say which starting point you use.

   Check: the final separation is smaller. Like charges need positive external work to move closer; unlike charges release energy.

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