Tutorial 1: Worked solutions

Electrical Engineering I · Module 92315 · Tutorial 1

From Fundamentals to AI Apocalypse · personal study guide

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Made by Thanate Amornjiraporn (Focus)

Use these first tasks to get comfortable with SI prefixes, vector arithmetic and coordinate systems. Try each part before opening its worked steps.

Task 1: Convert units

Rewrite each quantity using a suitable SI prefix while keeping its physical value unchanged.

Connected topic: Electric charge and field quantities (§1.1) use SI units; prefixes are powers of ten attached to those units.

1a–f: Choose the prefix

Try first: move the power of ten into a prefix and check that the leading number is in a convenient range.

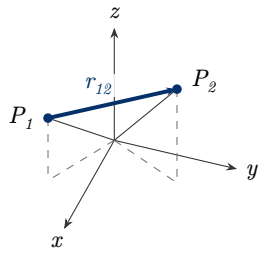
Answers: see "Task 1: Convert units" under Answers at the end.

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Task 2: Cartesian coordinate system

The points are $P_1 = (2, -1, 2)$ and $P_2 = (1, 2, 3)$. The coordinates have no stated unit, so the vector results are given in coordinate units.

Connected topic: Electric field as a vector (§1.3) uses components, magnitudes and directions in the same Cartesian basis.



Endpoint minus start. The position vectors of $\mathrm{P}_1 = (2, -1, 2)$ and $\mathrm{P}_2 = (1, 2, 3)$ run from the origin; the displacement r_{12} (blue) runs from P_1 to P_2, so $r_{12} = r_{\mathrm{P2}} - r_{\mathrm{P1}}$. The dashed lines drop each point onto the xy-plane.

2a: Displacement from P_1 to P_2

Try first: subtract the starting point coordinates from the ending point coordinates, component by component.

Answers: see "Task 2: Cartesian coordinate system" under Answers at the end.

2b: Magnitude of $\mathbf{r} = -\mathbf{e}_x + 3\mathbf{e}_y + \mathbf{e}_z$

Try first: square each perpendicular component, add them, then take the positive square root.

Answers: see "Task 2: Cartesian coordinate system" under Answers at the end.

2c: Unit vector in the direction of r

Try first: divide the vector by its magnitude; this preserves its direction and makes its length one.

Answers: see "Task 2: Cartesian coordinate system" under Answers at the end.

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Task 3: Coordinate systems

Use the standard spherical convention in this course: φ is the azimuth in the xy-plane from +x, and ϑ is the polar angle down from +z.

Connected topic: Flux and Gauss's law (§1.10) uses spherical coordinates for radial fields and spherical surfaces.

3a: Convert the position vector to spherical coordinates

Try first: find the distance from the origin, keep the given azimuth, and use the ratio of cylindrical radius to height to find the polar angle.

Answers: see "Task 3: Coordinate systems" under Answers at the end.

3b: Complete the spherical unit-vector table

Try first: remember that φ locates the projection in the xy-plane and ϑ measures down from +z. At the z-axis, azimuth is undefined.

Answers: see "Task 3: Coordinate systems" under Answers at the end.

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Answers

Task 1: Convert units

1a–f · Prefix conversions

1. **1a.** The decimal 0.000011 is 11×10^{-6}. The prefix micro (μ) means 10^{-6}:

$$C = 0.000011\,\mathrm{F} = 11 \times 10^{-6}\,\mathrm{F}$$

$$C = 11\,\mu\mathrm{F}$$

Check: 11 μF is 0.000011 F.

2. **1b.** Multiply the mantissas first: $47.47 \times 10^{5} = 4.747 \times 10^{6}$. Mega (M) means 10^{6}:

$$R = 4.747\,\mathrm{M\Omega}$$

Check: 4.747 MΩ = 4,747,000 Ω = 47.47 × 100,000 Ω.

3. **1c.** Combine the decimal factor and the given power: $0.062 \times 10^{-3} = 62 \times 10^{-6}$. Micro means 10^{-6}:

$$L = 62\,\mu\mathrm{H}$$

Check: 62 μH = 0.000062 H.

4. **1d.** Kilo (k) means 10^{3}, and $30{,}000 = 30 \times 10^{3}$:

$$U = 30\,\mathrm{kV}$$

Check: 30 kV = 30,000 V.

5. **1e.** $0.88 \times 10^{-4} = 8.8 \times 10^{-5} = 88 \times 10^{-6}$. Micro means 10^{-6}:

$$I = 88\,\mu\mathrm{A}$$

Check: 88 μA = 0.000088 A.

6. **1f.** $1265 \times 10^{6} = 1.265 \times 10^{9}$. Giga (G) means 10^{9}:

$$P = 1.265\,\mathrm{GW}$$

Check: 1.265 GW = 1,265,000,000 W.

Task 2: Cartesian coordinate system

2a · Displacement vector

1. A vector from point 1 to point 2 is the endpoint position minus the starting position:

$$r_{12} = r_{\mathrm{P2}} - r_{\mathrm{P1}}$$

2. Subtract each matching coordinate:

$$r_{12} = (1 - 2,\, 2 - (-1),\, 3 - 2)$$

$$r_{12} = (-1,\, 3,\, 1)$$

$$r_{12} = -e_{\mathrm{x}} + 3e_{\mathrm{y}} + e_{\mathrm{z}}$$

3. **Result:** the displacement is $(-1,\, 3,\, 1)$ coordinate units. The negative x-component means the motion is toward decreasing x; the y and z components are positive.

   Check: adding the displacement to P_1 gives $(2,\, -1,\, 2) + (-1,\, 3,\, 1) = (1,\, 2,\, 3)$, the stated endpoint.

Task 2: Cartesian coordinate system

2b · Vector magnitude

1. For Cartesian components, the length follows from the three-dimensional Pythagorean theorem:

$$|r| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

2. Here the components are $(-1,\, 3,\, 1)$, so:

$$|r| = \sqrt{(-1)^2 + 3^2 + 1^2}$$

$$|r| = \sqrt{11} \approx 3.32$$

   Result: $|r| = \sqrt{11}$ coordinate units.

   Check: a magnitude cannot be negative, and its square is $1 + 9 + 1 = 11$.

Task 2: Cartesian coordinate system

2c · Normalised direction

1. The unit vector is the original vector divided by its positive length:

$$e_{\mathrm{r}} = \frac{r}{|r|}$$

2. Divide the vector from 2a by its magnitude from 2b:

$$e_{\mathrm{r}} = \frac{-e_{\mathrm{x}} + 3e_{\mathrm{y}} + e_{\mathrm{z}}}{\sqrt{11}}$$

Result: $e_{\mathrm{r}} = (-1,\, 3,\, 1)/\sqrt{11}$.
Check: its length is $\sqrt{(1 + 9 + 1)/11} = 1$.

Task 3: Coordinate systems

3a · Cylindrical to spherical

1. The cylindrical components are $\rho = 5$, $z = 2$, and $\varphi = 45°$. The spherical radius is the distance from the origin:

$$r = \sqrt{\rho^2 + z^2} = \sqrt{25 + 4} = \sqrt{29}$$

2. ϑ is measured from +z, so its tangent is opposite over adjacent = ρ/z. The point is above the xy-plane, hence ϑ is in the first quadrant:

$$\vartheta = \arctan\left(\frac{\rho}{z}\right) = \arctan\left(\frac{5}{2}\right) \approx 68.2°$$

3. The azimuth remains the supplied angle, $\varphi = 45°$. State the spherical coordinates by name so the angle order is clear:

$$r = \sqrt{29}$$

$$\varphi = 45°$$

$$\vartheta \approx 68.2°$$

4. As a position vector, $r = re_{\mathrm{r}}$. Since $\vartheta = \arctan(5/2)$, $\sin\vartheta = 5/\sqrt{29}$ and $\cos\vartheta = 2/\sqrt{29}$, so the vector has no e_ϑ component:

$$r = \sqrt{29}\, e_{\mathrm{r}}$$

Check: its cylindrical projection is $r\sin\vartheta = 5$ and its z-component is $r\cos\vartheta = 2$.

Task 3: Coordinate systems

3b · Unit vectors and spherical angles

1. For a unit direction, take its projection onto each Cartesian basis vector:

$$e_r \cdot e_x = \sin\vartheta \cos\varphi$$

$$e_r \cdot e_y = \sin\vartheta \sin\varphi$$

$$e_r \cdot e_z = \cos\vartheta$$

   Evaluate these three components for each listed direction. At either pole ($\pm e_z$), φ can be chosen arbitrarily because the projection into the xy-plane has zero length.

2. The completed entries are:

Cartesian unit vector	φ	ϑ
e_z	any (the sheet's *)	0°
$-e_z$	any (the sheet's *)	180°
e_x	0°	90°
$-e_x$	180°	90°
e_y	90°	90°
$-e_y$	270°	90°
$(e_x + e_y)/\sqrt{2}$	45°	90°
$(e_x - e_z)/\sqrt{2}$	0°	135°
$(e_y + e_z)/\sqrt{2}$	90°	45°

3. For the last three rows, substitute the angles into the unit-vector formula. For example, $\varphi = 0°$ and $\vartheta = 135°$ gives $(\sin 135°, 0, \cos 135°) = (1/\sqrt{2}, 0, -1/\sqrt{2})$.

   Check: every listed Cartesian vector has unit length. Angles differing by 360° represent the same azimuth, so −90° is equivalent to 270°.

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