Tutorial 2: Worked solutions

Electrical Engineering I · Module 92315 · Tutorial 2

From Fundamentals to AI Apocalypse · personal study guide

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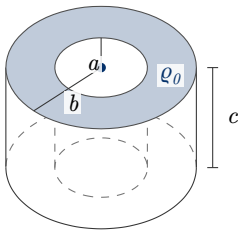
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These solutions build total charge by integrating the volume charge density over the region where it exists. Track the coordinate-dependent volume element and the density's support.

Task 4: Cylinder coordinates

The space charge density is constant, $\varrho_V = \varrho_0$, in the hollow cylindrical region $a \leq \rho \leq b$, $0 \leq \varphi \leq 2\pi$, $0 \leq z \leq c$, and zero outside. Assume $0 \leq a < b$ and $c > 0$.

Connected topic: Charge densities (§1.6): the total charge is the volume integral of the density. The cylindrical volume element is in Appendix A.5.



Only the wall is charged. The density ϱ_0 fills the region between the radii a and b (shaded on top) over the height c; the bore inside radius a is empty, so the radial integral starts at a.

4: Total charge in the hollow cylinder

Try first: write the cylindrical volume element, then integrate across the inner and outer radii, the full angle, and the height.

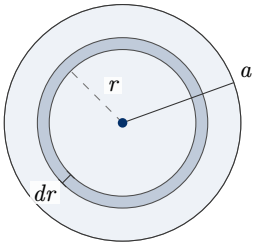
Answers: see "Task 4: Cylinder coordinates" under Answers at the end.

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Task 5: Spherical coordinates

The density varies radially as $\varrho_V(r) = \varrho_0 r^2/a^2$ inside a sphere of radius a and is zero for r > a. Assume a > 0.

Connected topic: Charge densities (§1.6): each thin spherical shell contributes its density times its volume. The spherical volume element is in Appendix A.5.



Add up thin shells. The shell at radius r and thickness dr (shaded) has volume $4\pi r^2$ dr and density $\varrho_0 r^2/a^2$; the outer shells are both larger and denser.

5: Total charge in the sphere

Try first: multiply the radial density by the spherical shell volume $4\pi r^2 dr$ and integrate from the centre to a.

Answers: see "Task 5: Spherical coordinates" under Answers at the end.

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Answers

Task 4: Cylinder coordinates

4 · Cylindrical volume integral

1. Total charge is the volume integral of volume charge density over the charged region. In cylindrical coordinates, a small volume is a wedge-shaped piece with volume ρ dρ dφ dz:

$$Q = \iiint_V \varrho_V \, \mathrm{d}V, \qquad \mathrm{d}V = \rho \, \mathrm{d}\rho \, \mathrm{d}\varphi \, \mathrm{d}z$$

2. Substitute the constant density and the stated limits. The inner radius a excludes the empty central cylinder:

$$Q = \varrho_0 \int_0^c \int_0^{2\pi} \int_a^b \rho \, \mathrm{d}\rho \, \mathrm{d}\varphi \, \mathrm{d}z$$

$$Q = \varrho_0 \int_0^c \int_0^{2\pi} \left[\frac{\rho^2}{2} \right]_a^b \mathrm{d}\varphi \, \mathrm{d}z$$

3. Evaluate the radial term, then the full revolution and cylinder height:

$$Q = \varrho_0 \, \frac{b^2 - a^2}{2} \, (2\pi) \, c$$

$$Q = \varrho_0 \, \pi (b^2 - a^2) c$$

The sign follows the sign of ϱ_0. **Check:** the factor $\pi(b^2 - a^2)c$ is exactly the hollow cylinder's volume, and ϱ_0 times volume has units C.

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Task 5: Spherical coordinates

5 · Spherical volume integral

1. For a spherically symmetric density, integrate over concentric shells. A shell at radius r has volume $4\pi r^2 dr$, and only radii 0 through a contain charge:

$$Q = \iiint_V \varrho_V \, dV = 4\pi \int_0^a \varrho_V(r) \, r^2 \, dr$$

2. Insert the given density. One factor r^2 comes from the shell area and another from ϱ_V:

$$Q = 4\pi \int_0^a \varrho_0 \frac{r^2}{a^2} \, r^2 \, dr$$

$$Q = \frac{4\pi\varrho_0}{a^2} \int_0^a r^4 \, dr$$

3. Integrate r^4 and apply both limits:

$$Q = \frac{4\pi\varrho_0}{a^2} \left[\frac{r^5}{5}\right]_0^a$$

$$Q = \frac{4\pi\varrho_0 \, a^3}{5}$$

The sign follows ϱ_0. **Check:** the average density is $3\varrho_0/5$, giving $(3\varrho_0/5)(4\pi a^3/3)$, the same result.

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