Tutorial 3: Worked solutions

Electrical Engineering I · Module 92315 · Tutorial 3

From Fundamentals to AI Apocalypse · personal study guide

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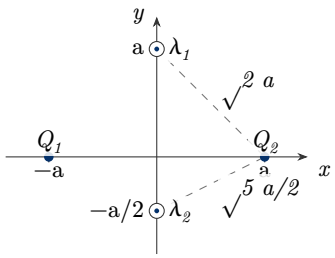
Resolve each interaction from its geometry, then add force vectors component by component. In Task 7, use Newton's third law to separate the known pair interaction from the supplied net forces.

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Task 6: Line and point charges

Use the shown xy cross-section: $Q_1 = (-a, 0)$, $Q_2 = (a, 0)$, the line λ_1 passes through $(0, a)$, and λ_2 passes through $(0, -a/2)$. Both lines run parallel to z. Take air as vacuum for the field formulas, as instructed, and assume $a > 0$.

Connected topic: Electric field strength (§1.3) gives Coulomb's field and vector superposition. The line-charge formula is applied with its radial direction converted to Cartesian components.



Distances to Q_2. The point charges sit on the x-axis at $\pm a$; the line charges run along z through $(0, a)$ and $(0, -a/2)$. The dashed lines are the perpendicular distances from each line to Q_2, $\sqrt{2}\ a$ and $\sqrt{5}\ a/2$, which set the line fields there.

6a: Field of Q_1 at Q_2

Try first: find the displacement from Q_1 to Q_2, then use Coulomb's field with that direction and distance.

Answers: see "Task 6: Line and point charges" under Answers at the end.

6b: Force of Q_1 on Q_2

Try first: multiply the field from 6a by the charge that experiences it, Q_2.

Answers: see "Task 6: Line and point charges" under Answers at the end.

6c: Field of λ_1 and λ_2 at Q_2

Try first: form the transverse displacement from each line to Q_2, normalize it, and add the two line fields.

Answers: see "Task 6: Line and point charges" under Answers at the end.

6d: Force of the lines on Q_2

Try first: use the field sum from 6c and multiply by Q_2.

Answers: see "Task 6: Line and point charges" under Answers at the end.

6e: Total force on Q_2

Try first: add the point-charge force from 6b to the line-charge force from 6d, keeping x and y components separate.

Answers: see "Task 6: Line and point charges" under Answers at the end.

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6f: Condition for zero total force

Try first: set both components from 6e to zero and solve the y equation before the x equation.

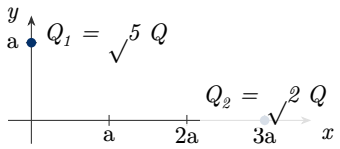
Answers: see "Task 6: Line and point charges" under Answers at the end.

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Task 7: Point charge

Take $Q > 0$ as the reference scale. The figure places $Q_1 = \sqrt{5}\,Q$ at (0, a) and $Q_2 = \sqrt{2}\,Q$ at (3a, 0). The supplied net forces on Q_1 and Q_2 are $F_1 = F_0[-(3 + 5\sqrt{5})e_x + e_y]$ and $F_2 = F_0(13e_x - 11e_y)$, with $F_0 = Q^2/(40\pi\varepsilon_0 a^2)$. The charges are in vacuum and $a > 0$; F_0 is a force scale measured in newtons.

Connected topic: Electric field strength (§1.3) connects Coulomb force, displacement direction and vector superposition.



The two known charges. $Q_1 = \sqrt{5}\,Q$ sits at $(0, a)$ and $Q_2 = \sqrt{2}\,Q$ at $(3a, 0)$. The third charge is not drawn: its position is what 7e finds from the directions of the forces.

7a: Force F_{12} of Q_2 on Q_1

Try first: find the vector from Q_2 to Q_1 and substitute both charges and their separation into Coulomb's law.

Answers: see "Task 7: Point charge" under Answers at the end.

7b: Force F_{21} of Q_1 on Q_2

Try first: use Newton's third law for the same pair.

Answers: see "Task 7: Point charge" under Answers at the end.

7c: Force F_{13} of Q_3 on Q_1

Try first: subtract the known Q_2 contribution from the supplied net force on Q_1.

Answers: see "Task 7: Point charge" under Answers at the end.

7d: Force F_{23} of Q_3 on Q_2

Try first: add F_{12} to the net force on Q_2, using $F_{21} = -F_{12}$.

Answers: see "Task 7: Point charge" under Answers at the end.

7e: Coordinates of Q_3

Try first: intersect the horizontal line through Q_1 with the diagonal direction line through Q_2.

Answers: see "Task 7: Point charge" under Answers at the end.

7f: Charge ratio Q_3/Q

Try first: use the magnitude of F_{13}, the separation 2a, and $Q_1 = \sqrt{5}\,Q$.

Answers: see "Task 7: Point charge" under Answers at the end.

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7g: Total force F_3 on Q_3

Try first: reverse F_{13} and F_{23} by Newton's third law, then add them.

Answers: see "Task 7: Point charge" under Answers at the end.

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Answers

Task 6: Line and point charges

6a · Point-charge field

1. The source is at x = −a and the field point at x = a, so the displacement points along +x and has length 2a:

$$R_{21} = 2a\, e_{\mathrm{x}}, \qquad |R_{21}| = 2a$$

2. Coulomb's field is the source charge times the radial unit vector divided by $4\pi\varepsilon_0 R^2$:

$$E_{\mathrm{Q1}}(Q_2) = \frac{Q_1}{4\pi\varepsilon_0\,|R_{21}|^2}\, e_{\mathrm{x}}$$

$$E_{\mathrm{Q1}}(Q_2) = \frac{Q_1}{16\pi\varepsilon_0\, a^2}\, e_{\mathrm{x}}$$

The sign of Q_1 sets whether the field points +x or −x. The $1/(2a)^2$ factor is easy to miss.

Task 6: Line and point charges

6b · Point-charge force

1. The electric force on a charge is its charge times the electric field at its position:

$$F_{12} = Q_2\, E_{\mathrm{Q1}}(Q_2)$$

2. Substitute the result from 6a:

$$F_{12} = \frac{Q_1\, Q_2}{16\pi\varepsilon_0\, a^2}\, e_{\mathrm{x}}$$

Like signs give a force toward +x on Q_2; opposite signs reverse it. Its units are newtons because charge times field is C·N/C.

Task 6: Line and point charges

6c · Infinite-line fields

1. For λ_1, the displacement from its axis (0, a) to Q_2 (a, 0) is (a, −a). Its radius is $\sqrt{2}$ a and its radial unit vector is $(e_x - e_y)/\sqrt{2}$:

$$E_{\lambda 1} = \frac{\lambda_1}{2\pi\varepsilon_0 \sqrt{2}\, a} \cdot \frac{e_x - e_y}{\sqrt{2}} = \frac{\lambda_1}{4\pi\varepsilon_0 a}(e_x - e_y)$$

2. For λ_2, the displacement is (a, a/2), with radius $\sqrt{5}$ a/2 and unit direction $(2e_x + e_y)/\sqrt{5}$:

$$E_{\lambda 2} = \frac{\lambda_2}{\pi\varepsilon_0 \sqrt{5}\, a} \cdot \frac{2e_x + e_y}{\sqrt{5}} = \frac{\lambda_2}{5\pi\varepsilon_0 a}(2e_x + e_y)$$

3. Add the x and y components separately:

$$E_{\text{lines}} = \frac{1}{\pi\varepsilon_0 a}\left[\left(\frac{\lambda_1}{4} + \frac{2\lambda_2}{5}\right) e_x + \left(-\frac{\lambda_1}{4} + \frac{\lambda_2}{5}\right) e_y\right]$$

Check the second line's distance is $a\sqrt{5}/2$, not $a\sqrt{5}$; its source is only a/2 below the x-axis. Each field component has units N/C.

Task 6: Line and point charges

6d · Force from both lines

1. Both line fields act at Q_2, so their combined force is Q_2 times their vector sum:

$$F_{\text{lines},2} = Q_2\, E_{\text{lines}}$$

2. Therefore:

$$F_{\text{lines},2} = \frac{Q_2}{\pi\varepsilon_0 a}\left[\left(\frac{\lambda_1}{4} + \frac{2\lambda_2}{5}\right) e_x + \left(-\frac{\lambda_1}{4} + \frac{\lambda_2}{5}\right) e_y\right]$$

A negative component indicates the corresponding force points along the negative axis; the force components are in newtons.

Task 6: Line and point charges

6e · Superposition of all forces

1. Electrostatic forces add linearly, so the total is the point-charge contribution plus the two-line contribution:

$$F_2 = F_{12} + F_{\text{lines},2}$$

2. Collect the coefficients of each Cartesian direction:

$$F_2 = \left[\frac{Q_1\,Q_2}{16\pi\varepsilon_0\,a^2} + \frac{Q_2}{\pi\varepsilon_0\,a}\left(\frac{\lambda_1}{4} + \frac{2\lambda_2}{5}\right)\right] e_{\text{x}} + \frac{Q_2}{\pi\varepsilon_0\,a}\left(-\frac{\lambda_1}{4} + \frac{\lambda_2}{5}\right) e_{\text{y}}$$

Check: if both line densities vanish, this reduces to the Coulomb force in 6b.

Task 6: Line and point charges

6f · Balance the two components

1. Q_2 is nonzero, so the y-component vanishes when:

$$-\frac{\lambda_1}{4} + \frac{\lambda_2}{5} = 0 \qquad \Rightarrow \qquad \lambda_2 = \frac{5\lambda_1}{4}$$

2. Insert this relation into the x-component, including the field from Q_1:

$$\frac{Q_1}{16a^2} + \frac{\lambda_1}{4a} + \frac{2}{5a}\cdot\frac{5\lambda_1}{4} = 0$$

$$\frac{Q_1}{16a^2} + \frac{3\lambda_1}{4a} = 0 \qquad \Rightarrow \qquad Q_1 = -12a\lambda_1$$

3. Thus the required condition is $\boldsymbol{\lambda_2 = 5\lambda_1/4}$ **and** $\boldsymbol{Q_1 = -12a\lambda_1}$ (equivalently $\lambda_1 = -Q_1/(12a)$, $\lambda_2 = -5Q_1/(48a)$). These signs make the line forces cancel in y and balance Q_1's x contribution.

Task 7: Point charge

7a · First known pair force

1. The displacement from Q_2 at (3a, 0) to Q_1 at (0, a) is (−3a, a), and the distance is $\sqrt{10}$ a:

$$R_{12} = a(-3e_x + e_y), \qquad |R_{12}| = \sqrt{10}\,a$$

2. Coulomb force on Q_1 from Q_2 is kQ_1Q_2 times the unit displacement divided by R^2. Here $Q_1Q_2 = \sqrt{10}\ Q^2$:

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1\,Q_2}{|R_{12}|^3}\,R_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{10}\,Q^2}{10\sqrt{10}\,a^3}\,a(-3e_x + e_y)$$

$$F_{12} = \frac{Q^2}{40\pi\varepsilon_0\,a^2}(-3e_x + e_y) = F_0(-3e_x + e_y)$$

The displacement points from the source toward Q_1, so for like charges this force points up and to the left, away from Q_2.

Task 7: Point charge

7b · Equal and opposite force

1. Forces between a pair have equal magnitude and opposite direction:

$$F_{21} = -F_{12}$$

2. Therefore:

$$F_{21} = F_0(3e_x - e_y)$$

The two pair forces sum to zero when applied to both charges together.

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Task 7: Point charge

7c · Isolate the remaining force

1. The given resultant on Q_1 is $F_1 = F_{12} + F_{13}$, so rearrange to isolate the force from Q_3:

$$F_{13} = F_1 - F_{12}$$

2. Subtract the components (the $+e_y$ terms cancel):

$$F_{13} = F_0[-(3 + 5\sqrt{5})e_x + e_y - (-3e_x + e_y)]$$

$$F_{13} = -5\sqrt{5}\,F_0\,e_x$$

Its purely horizontal direction places Q_3 on the horizontal line through Q_1.

Task 7: Point charge

7d · Second remaining pair force

1. The given resultant on Q_2 is $F_2 = F_{21} + F_{23}$. Since $F_{21} = -F_{12}$:

$$F_{23} = F_2 - F_{21} = F_2 + F_{12}$$

2. Combine components:

$$F_{23} = F_0[(13 - 3)e_x + (-11 + 1)e_y]$$

$$F_{23} = 10F_0(e_x - e_y)$$

Its direction line is diagonal, with equal x and negative y components.

Task 7: Point charge

7e · Locate the third charge

1. F_{13} has no y component, so the line joining Q_1 and Q_3 is horizontal. Since $y_1 = a$, this gives $y_3 = a$.

2. F_{23} is parallel to $(1, -1)$. The line through $Q_2 = (3a, 0)$ in that direction has form $(3a, 0) + t(1, -1)$. Its y-coordinate equals a when $t = -a$:

$$(x_3, y_3) = (3a, 0) - a(1, -1) = (2a, a)$$

3. **Result:** Q_3 is at (2a, a). This position also gives $Q_1Q_3 = 2a$ and $Q_2Q_3 = \sqrt{2}\,a$.

Task 7: Point charge

7f · Find the third charge

1. The force magnitude between Q_1 and Q_3 is $5\sqrt{5}\ F_0$. Coulomb's law at separation 2a gives:

$$5\sqrt{5}\,F_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{5}\,Q\,|Q_3|}{(2a)^2}$$

2. Insert $F_0 = Q^2/(40\pi\varepsilon_0 a^2)$ and cancel common positive factors:

$$\frac{5\sqrt{5}\,Q^2}{40\pi\varepsilon_0\,a^2} = \frac{\sqrt{5}\,Q\,|Q_3|}{16\pi\varepsilon_0\,a^2}$$

$$|Q_3| = 2Q$$

3. F_{13} points from Q_3 toward Q_1, and Q_3 is to the right of Q_1; the force is repulsive. Thus Q_3 has the same sign as Q_1, and $\mathbf{Q_3/Q = 2}$.

Task 7: Point charge

7g · Resultant on Q_3

1. The force on Q_3 from Q_1 is opposite the force of Q_3 on Q_1; the same reversal holds for the Q_2 pair:

$$F_{31} = -F_{13} = 5\sqrt{5}\,F_0\,e_x, \qquad F_{32} = -F_{23} = -10F_0\,e_x + 10F_0\,e_y$$

2. Now add the two forces acting on Q_3:

$$F_3 = F_0[(5\sqrt{5} - 10)e_x + 10e_y]$$

Q_2, below and to the right of Q_3, pushes it up and to the left; Q_1, to its left, pushes it to the right. The two x parts nearly cancel, leaving a small positive x component and a positive y component. Numerically, $5\sqrt{5} - 10 \approx 1.18$.

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