Tutorial 6: Worked solutions

Electrical Engineering I · Module 92315 · Tutorial 6

From Fundamentals to AI Apocalypse · personal study guide

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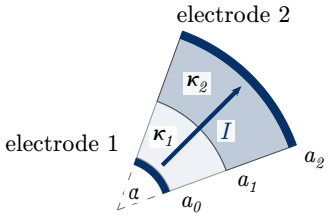
Track the conserved current through each cylindrical sector, use local Ohm's law in each material, then integrate the radial field across both layers.

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Task 10: Stationary electric flow field

The conductor is an annular sector with axial height h and angular span α. An ideal inner electrode at radius a_0 feeds current through material $\varkappa_1$ from a_0 to a_1, then through material $\varkappa_2$ from a_1 to a_2 to the outer electrode. Take positive I and U_{12} in the outward direction from electrode 1 to electrode 2, as drawn; use α in radians.

Connected topic: Current density (§2.1.2) and local Ohm's law (§2.2.2) determine the radial fields; resistance (§2.2.4) follows by integrating through the layered path.



Seen from above. The conductor is an annular sector of angle α and height h: $\varkappa_1$ from a_0 to a_1, $\varkappa_2$ from a_1 to a_2, between the ideal electrodes 1 (inner) and 2 (outer). The current runs radially outward; at radius ρ it crosses the area $\alpha\rho h$.

10a: Set up the current-density field

Try first: use charge conservation at steady state and the normal-current boundary condition at the $\varkappa_1/\varkappa_2$ interface.

Answers: see "Task 10: Stationary electric flow field" under Answers at the end.

10b: Find J as a function of I

Try first: calculate the area normal to radial flow at radius ρ, then divide the total current by that area.

Answers: see "Task 10: Stationary electric flow field" under Answers at the end.

10c: Find the electric field strength

Try first: apply $J = \varkappa E$ separately in each radial layer.

Answers: see "Task 10: Stationary electric flow field" under Answers at the end.

10d: Find $U_{12}(I)$

Try first: integrate the radial field along the current path, changing conductivity at the material boundary.

Answers: see "Task 10: Stationary electric flow field" under Answers at the end.

10e: Find the resistance

Try first: divide the voltage from 10d by the current, or add the two layer resistances in series.

Answers: see "Task 10: Stationary electric flow field" under Answers at the end.

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Answers

Task 10: Stationary electric flow field

10a · Continuity across the layers

1. In a stationary flow, charge does not accumulate in the conductor, so the net current through each coaxial sector surface is the same. At the interface $\rho = a_1$, the normal is e_ρ: its normal current component is continuous, and the tangential electric-field components (zero here) are continuous too.
2. The current flows radially, normal to the cylindrical interface. Therefore $J_{\rho,1} = J_{\rho,2}$. Local Ohm's law $J = \varkappa E$ means the electric field generally changes at the interface:

$$\frac{E_{\rho,2}}{E_{\rho,1}} = \frac{\kappa_1}{\kappa_2}$$

3. The ideal electrodes have $\varkappa \to \infty$, so their internal electric field is zero while they remain equipotential. Neglect edge spreading at the ends of the sector; the current in the two finite-conductivity layers is radial.

Task 10: Stationary electric flow field

10b · Current through a cylindrical sector

1. At radius ρ, the curved surface spans arc length $\alpha\,\rho$ and axial height h, so its area normal to the flow is:

$$A(\rho) = \alpha \rho h$$

2. By symmetry the radial current density is uniform over this surface, and $I = \int J \cdot \mathrm{d}A = J_\rho A(\rho)$:

$$J(\rho) = \frac{I}{\alpha h \rho}\, e_\rho, \qquad a_0 < \rho < a_2$$

The same expression holds on both sides of a_1. It falls as $1/\rho$ because the current-carrying area grows with radius.

Task 10: Stationary electric flow field

10c · Piecewise Ohm's law

1. In the first layer, $a_0 < \varrho < a_1$, divide the current density by $\varkappa_1$:

$$E_1(\rho) = \frac{I}{\alpha h \kappa_1 \rho} e_\rho$$

2. In the second layer, $a_1 < \varrho < a_2$, use $\varkappa_2$ instead:

$$E_2(\rho) = \frac{I}{\alpha h \kappa_2 \rho} e_\rho$$

At the interface the normal current remains continuous, while E jumps by the conductivity ratio found in 10a. The direction is outward for positive I.

Task 10: Stationary electric flow field

10d · Voltage drop across both layers

1. With U_{12} defined as electrode-1 potential minus electrode-2 potential, it equals the outward field integral from a_0 to a_2. Split the integral where $\varkappa$ changes:

$$U_{12} = \int_{a_0}^{a_1} E_{1,\rho}\,\mathrm{d}\rho + \int_{a_1}^{a_2} E_{2,\rho}\,\mathrm{d}\rho$$

2. Substitute the fields and integrate $\mathrm{d}\varrho/\varrho$ in each interval:

$$U_{12}(I) = \frac{I}{\alpha h \kappa_1} \ln\left(\frac{a_1}{a_0}\right) + \frac{I}{\alpha h \kappa_2} \ln\left(\frac{a_2}{a_1}\right)$$

The voltage is positive for outward positive current. Each logarithm is dimensionless.

Task 10: Stationary electric flow field

10e · Series resistance of radial layers

1. Resistance is $R = U_{12}/I$. Since the current passes through $\varkappa_1$ and $\varkappa_2$ sequentially, their radial resistances add:

$$R_1 = \frac{1}{\alpha h \kappa_1} \ln\left(\frac{a_1}{a_0}\right), \qquad R_2 = \frac{1}{\alpha h \kappa_2} \ln\left(\frac{a_2}{a_1}\right)$$

2. Therefore:

$$R = \frac{1}{\alpha h}\left[\frac{1}{\kappa_1} \ln\left(\frac{a_1}{a_0}\right) + \frac{1}{\kappa_2} \ln\left(\frac{a_2}{a_1}\right)\right]$$

The units are ohms: $1/(\varkappa h)$ has units Ω, and the angle and logarithms are dimensionless. If $\varkappa_1 = \varkappa_2 = \varkappa$, the logs combine to $\ln(a_2/a_0)$, a useful check.

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